Issues in Modeling and Adjusting Calendar Effects in Economic Time Series

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Abstract

The effectiveness of alternate models for estimating trading day and moving holiday effects in economic time series are examined. Several alternative approaches to modeling Easter holiday effects will be examined, including a method suggested by the Australian Bureau of Statistics that includes a linear effect. In addition, a more parsimonious technique for modeling trading day variation will be examined by applying the day-of-week constraints from the weekday/weekend trading day contrast regressor found in TRAMO and X-12-ARIMA to stock trading day.

Keywords: moving holiday effects, forecast revisions, seasonal adjustment, likelihood statistics

1 Introduction

This study examines the effectiveness of some alternate models for estimating working day and moving holiday effects in economic time series. It is focused on two areas of interest: alternate models for Easter holiday effects, and constrained stock trading day regressors.

Several approaches to modeling Easter holiday effects in U. S. Retail Sales series are examined, including adaptations of Easter regressors studied by the Australian Bureau of Statistics (see Zhang, McLaren, and Leung (2003)), as well as new models that break the Easter model into separate weekday and weekend effects.

In addition, a more parsimonious technique for modeling trading day variation is examined by developing a weekday-weekend constrained version of the stock trading day regressor as described in Findley and Monsell (2007), which imposes flow day-of-week effect constraints upon the day-of-week effect component of the stock trading day model of Bell (1984) used in X-12-ARIMA. These constrained trading day variables will be applied to industrial inventory series.

2 Alternate Models for Easter

The most common moving holiday effect for monthly flow series found in U. S. economic series is the Easter effect. For many retail sales series, levels of sales are elevated in the period just before the Easter holiday (which varies between March 22 and April 25). Because of this, X-12-ARIMA has long had a built in regressor corresponding to the Easter holiday.

The current Easter regressor within X-12-ARIMA assumes that the level of activity changes on the $w-$th day before the holiday for a specified w, and remains at the new level until the day before the holiday. For a given effect window w, the Easter regressor is generated as

$$E(w,t) = \frac{n_{w,t}}{w} - \mu_{w,t} \tag{1}$$

where $n_{w,t}$ is the number of the w days before Easter fall in month t, and $\mu_{w,t}$ are the "long-run" monthly (or quarterly) means of the first part of the $E(w,t)$ equation (corresponding to the first 400 year period of the Gregorian calendar, 1583-1982)[1].

There have been two main critiques of this type of Easter regressor. One is that assuming the level of activity is elevated by a constant level for the w days before Easter is unrealistic. A better measure of sales activity before Easter would allow for a linear increase in activity before the Easter holiday. Zhang, McLaren, and Leung (2003) studied the performance of an Easter regressor that could handle such an effect. It was found to perform about as well as the current Easter regressor in Equation (1). An Easter regressor similar to the quadratic regressor of Zhang, McLaren, and Leung (2003) will be applied to retail sales series.

Some analysts have suggested that an allowance be made for an assumed decline in activity during Good Friday and the Easter holiday itself. Their argument is that not including such an effect leads to over-adjustment of the Easter holiday effect. This could be particularly important when Easter occurs late in March.

Zhang, McLaren, and Leung (2003) developed an alternate regressor for a similar effect in Australian series. Their model consisted of two regressors, one which modeled an assumed increase in the level of the series before Good Friday, and another to handle the period between Good Friday and Easter Monday, a national holiday in Australia. Again, alternate regressors similar to the Australian model will be used for US series.

2.1 2-Part Easter Regressor

The first alternate form for modeling the Easter effect assumes that one can break the Easter effect into two parts: a pre-Easter effect from the $w-th$ day before Easter to the day before Good Friday, and an Easter Holiday effect starting on Good Friday

[1]Note that the long term mean is in the same units as the ratio of days in the effect window w in the first part of Equation (1).

and lasting until Easter. This takes the form of two regressors - a pre-holiday effect (where sales are expected to be elevated) and an effect for the duration of the holiday (where sales are expected to decline). These regressors will be referred to as **two-part** Easter regressors throughout this paper.

Let n_t^{BE} be the number of days from the w-th day before Easter to the day before Good Friday that fall in month t, and n_t^{DE} be the number of days between Good Friday and Easter (inclusive) that fall in month t. The pre-holiday regressor is generated as

$$BE(w,t) = \frac{n_t^{BE}}{w-3} - \mu_{w,t}^{BE} \tag{2}$$

and the effect during the holiday is generated as

$$DE(t) = \frac{n_t^{DE}}{3} - \mu_t^{DE} \tag{3}$$

where again, $\mu_{w,t}^{BE}$ and $\mu_{w,t}^{DE}$ are the "long-run" monthly (or quarterly) means used to center the respective Easter regressors.

2.2 Linear Easter Regressor

The next regressor assumes the level of activity before Easter increases linearly before the holiday. This increase in level starts at the $w-th$ day before the holiday for a specified w, and increases each day until the day before the holiday.

For a given effect window w for monthly series, let $n_{March,y}$ be the number of days before Easter falling in March (or the first quarter) for year y. The value for the **linear Easter regressor** for March is taken to be

$$LE(w, March, y) = \frac{n_{March,y}^2}{w^2} - \mu_{w,March}^{LE} \tag{4}$$

and the value for April (or the second quarter) is

$$LE(w, April, y) = \left(1 - \left(\frac{n_{March,y}^2}{w^2}\right)\right) - \mu_{w,April}^{LE} \tag{5}$$

where $\mu_{w,March}^{LE}$ and $\mu_{w,April}^{LE}$ are the "long-run" monthly (or quarterly) means of the first part of equations (4) and (5). Values for all other months (or quarters) are assumed to be zero.

Note that we can construct **two-part linear Easter regressors** analogous to those in Section 2.1. In this case, let $n_{March,y}^{BE}$ be the number of days in the period starting w days before Easter to the day before Good Friday that fall in March. Then in the same way as in equation (4), we have

$$LBE(w, March, y) = \frac{(n_{March,y}^{BE})^2}{w^2} - \mu_{w,March}^{LE} \tag{6}$$

for the March (or first quarter) value of the pre-Good Friday regressor. The value of this regressor for April (or the second quarter) follows from equation (5) as

$$LBE(w, April, y) = \left(1 - \left(\frac{(n_{March,y}^{BE})^2}{w^2}\right)\right) - \mu_{w,April}^{LE} \tag{7}$$

with the values for the remaining months (or quarters) being zero.

The second regressor for this two-part Easter regressor will be the same Easter duration regressor that was defined earlier in equation (3).

2.3 Weekend-Weekday Easter Regressors

The final proposed Easter model is one that assumes that the change in the level of activity for weekend days (Friday, Saturday, and Sunday) is different than the change for the weekdays leading up to Easter. This model will pool the effect for the weekend and weekdays, so that there is a single regressor for the weekend effect and a single regressor for the weekday effect.

Let $n_{we,t}$ be the number days in month t that fall on a Friday, Saturday, or Sunday in the period 16 days before Easter, inclusive, and $n_{wd,t}$ be the number days in month t that fall on a Monday, Tuesday, Wednesday or Thursday in the period 16 days before Easter. Then the Weekend-Weekday Easter regressors are

$$WE(t) = \frac{n_{we,t}}{8} - \mu_{we} \tag{8}$$

$$WD(t) = \frac{n_{wd,t}}{8} - \mu_{wd} \tag{9}$$

where μ_{we} and μ_{wd} are the "long-run" monthly (or quarterly) means of the first part of equations (8) and (9) corresponding to the first 400 year period of the Gregorian calendar, 1583-1982.

Note that this definition excludes Easter and includes Good Friday in the weekend regressor defined in equation (8). Another configuration would create a third regressor that would try to capture an effect for Good Friday and Easter, a **Weekend-Weekday-Easter regressor**.

Let $n_{wee,t}$ be the number of days in month t that fall on a Friday, Saturday, or Sunday in the period 16 days before Easter excluding Friday, inclusive, $n_{wde,t}$ be the number of days in month t that fall on a Monday, Tuesday, Wednesday or Thursday in the period 16 days before Easter, and $n_{gfe,t}$ be the number of times Good Friday and Easter occur in month t. Then the Weekend-Weekday-Easter regressors are

$$WEE(t) = \frac{n_{wee,t}}{7} - \mu_{wee} \tag{10}$$

$$WDE(t) = \frac{n_{wde,t}}{8} - \mu_{wde} \tag{11}$$

$$GFE(t) = \frac{n_{gfe,t}}{2} - \mu_{gfe} \tag{12}$$

where μ_{wee}, μ_{wde}, and μ_{gfe} are the "long-run" monthly (or quarterly) means of the first part of equations (10), (11) and (12) corresponding to the first 400 year period of the Gregorian calendar, 1583-1982. Note that the regressor produced in equations (9) and (11) are equivalent.

2.4 Modeling Diagnostics Used to Choose Between Easter Models

An earlier study, Findley and Soukup (2000), shows how comparing AIC values and analyzing graphs of out-of-sample forecast errors can be used to determine if regARIMA models should include moving holiday terms. The same diagnostics will be used in this study, testing each of the different alternate models in sequence to determine which Easter model should be used.

The first utilizes likelihood-based model selection criterion by comparing the values of AICC, a version of Akaike's Information Criterion also called the F-corrected AIC which contains a correction for small sample size. Suppose the number of estimated parameters in the model, including the white noise variance, is n_p. If after applying the model's differencing and seasonal differencing operations, there are N data, and if the estimated maximum value of the exact log likelihood function of the model is denoted L_N, then the formula for the AICC criteria is:

$$AICC_N = -2L_N + 2n_p\left(1 - \frac{n_p+1}{N}\right)^{-1} \tag{13}$$

Among competing models for a given times series, the model with the smallest AICC value is the model preferred by the criterion. For more information on AICC, see Hurvich and Tsai (1989).

Another method for determining which Easter regressor to choose is to examine out of sample forecast error plots available from X-12-Graph (see Hood (2002)). X-12-ARIMA's `history` spec is used to obtain differences of the accumulating sums of squared forecast errors between the competing models for forecast leads of interest (in this case, 1 and 12). Accumulating forecast errors are computed for two competing models, and the difference between the values for the two models are graphed.

The first model is preferred if the direction of the accumulating differences is predominantly downward. We assume in this case the forecast errors are predominantly smaller for the first model.

If the direction of the accumulating differences is predominantly upward, then we assume that the forecast errors are predominantly larger for the first model, and prefer the second model.

Often the forecast error differences do not appear to be going in any particular direction. Other times the direction of the accumulating differences goes in one direction for one forecast lag, and another direction for the other. In these cases, the forecast error plots are inconclusive, and we cannot declare a preferred model.

2.5 Application to Retail Sales Series

Currently 17 retail sales series published monthly by the Census Bureau are adjusted for Easter effects (among other calendar effects). Table 1 gives the names and descriptions of these series.

Series	Description of Retail Sales Series
s0b44510	Grocery
s0b445X0	Miscellaneous Food and Beverage
s0b44520	Miscellaneous Food
s0b44530	Beverage and Liquor
s0b44600	Health and personal care
s0b44611	Pharmacies
s0b44811	Men's Clothing
s0b44812	Women's Clothing
s0b4481L	Miscellaneous and Luggage
s0b4481Y	Miscellaneous Apparel
s0b45210	Department Stores (Excluding leased departments)
s0b45212	Discount Department Stores (Excluding leased departments)
s0b45231	Convenience Stores (Excluding leased departments)
s0b45291	Super stores
s0b45299	Miscellaneous General Merchandise
s0b45310	Florists

Table 1: Descriptions of Retail Sales series modeled using the Easter[8] regressor (Source: U.S. Census Bureau).

The Easter adjustments are generated from regARIMA models fit to the series. The `easter[8]` regressor is used for all the series (an Easter regressor where the window w in equation (1) is 8). The data used to model the series will start at January 1995 for each of the series.

The regARIMA model used by the Census Bureau to produce the monthly seasonal adjustments is the model used for this study. The only part of the model that is changed throughout this analysis is the model for Easter used (or not used).

Note that automatic outlier identification will not be performed during the analysis. It would bias the model comparisons if different outliers were identified for different Easter models.

2.5.1 AICC Results

The first comparison is between four models:

- the regARIMA model for the series with no Easter regressor
- the regARIMA model with an `easter[1]` regressor
- the regARIMA model with an `easter[8]` regressor
- the regARIMA model with an `easter[15]` regressor

Note that these are the choices for Easter regressors used in the `aictest` argument for Easter. The model that the minimum AICC criterion chooses most often in Table 2 is the model with the `easter[8]` regressor, the Easter regressor currently used by the Census Bureau for adjusting these series.

The third columns of Table 2 gives the difference between the AICC of the preferred Easter model and that of the model with no Easter regressor. Usually, an AICC difference of 1 is enough to select between a pair of models; models with AICC difference less than one (in absolute value) cannot be distinguished from one another.

Table 2 also gives the choice of the two-part Easter regression model defined in Section 2.1. Note that in only one case does the two-part Easter model have a better AICC than the model AICC chose for the current Easter models - Retail Sales of Miscellaneous Food and Beverage (s0b445X0). Only seven of the series had an Easter duration regressor for one of the two-part Easter models with a significant t-value (> 2.00). For a majority of the series, the presence of the extra regressor does not significantly improve the model.

Tables 3 gives the model choice due to AICC for the linear Easter regression model defined in Section 2.2. Note again that the differences between the AICCs for the preferred linear Easter model and the model with no Easter indicates that the linear Easter should be in the model.

Also, there are six cases out of the sixteen series where the two-part linear Easter model has a better AICC than the linear Easter model AICC chose in Table 3. Again, for most of the series, the extra regressor for the Good Friday to Easter effect does not improve the model for most of the cases.

Table 4 gives the final choice of Easter regressor from all the sets of Easter models, including the Weekend-Weekday Easter model defined in Section 2.3, giving the preference between the linear Easter regressor and their current Easter regressors from X-12-ARIMA as well as the Easter model that performs best overall.

First, there does not seem to a discernable difference in the likelihood statistics between the linear Easter regressor and the current Easter regressor. There are only two cases where the linear Easter regressor is the preferred model and the AICC difference is greater than one.

The Easter model that is selected most often is the Weekend-Weekday Easter model, and the `easter[8]` regressor used currently is not selected at all. When we examine the AICC differences between models with the current regressor and models with the Easter regressor(s) with the lowest AICC, there is a greater preference for the Weekend-Weekday regressors; many differences are small for the linear Easter regressors when it is preferred.

2.5.2 Forecast Error Plot

Examining the forecast error plots, two questions need to be addressed:

- Does using an alternate Easter model offer an improvement over the current methodology?
- Does using an alternate Easter model offer an improvement over not using an Easter regressor at all?

For the first, the forecast plots offer some support for the conclusions given by the AICCs - for one thing, the differences between using the `easter[8]` and linear Easter regressors appear to be quite small. The graph for Retails Sales

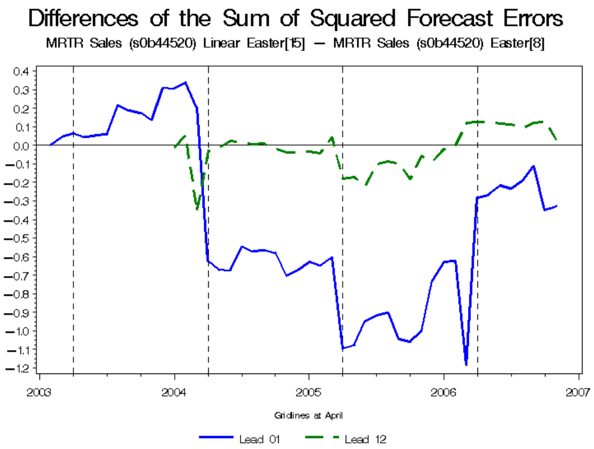

Figure 1: Forecast Error plot comparing forecasts from models with Easter[8] and Linear Easter[15] regressors for Retail Sales of Miscellaneous Food (s0b44520) (source: U. S. Census Bureau).

of Miscellaneous Food (s0b44520) in figure 1 is an example of this; the differences between the evolving sum of squared forecast errors are extremely small, and a consistent pattern in one direction or the other never develops.

A clearer indication of improvement is given for the Weekend-Weekday regressor over the `easter[8]` regressor in the forecast error plot of Retail Sales of Miscellaneous Apparel and Luggage (s0b4481L) given in figure 2. The downward slope and position of the line overall shows an improvement in the forecasts using the Weekend-Weekday Easter variables (although there is movement upward in the last two years).

In addition, some of the models with a preference for `easter[1]` showed improvement over `easter[8]` as well.

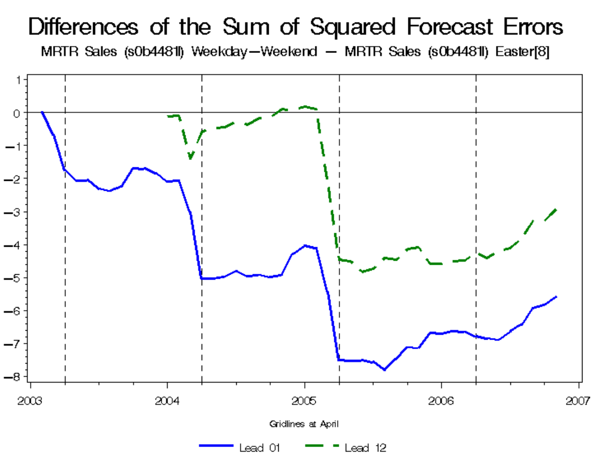

Figure 2: Forecast Error plot comparing forecasts from models with different Easter regressors for Retail Sales of Miscellaneous Apparel and Luggage (s0b4491L) (source: U. S. Census Bureau).

Series	Choice of Easter regressor	AICC Difference Best Easter vs. None	Choice of 2-Part Easter	Choice of 2-Part vs. Current
s0b44510	easter[8]	-57.4456	2-Part e[15]	easter[8]
s0b44520	easter[8]	-95.5329	2-Part e[8]	easter[8]
s0b44530	easter[1]	-6.2961	2-Part e[8]	easter[1]
s0b445X0	easter[8]	-66.2861	2-Part e[8]	2-Part easter[8]
s0b44600	easter[8]	-19.6923	2-Part e[8]	easter[8]
s0b44611	easter[8]	-20.8301	2-Part e[8]	easter[8]
s0b44811	easter[1]	-26.2221	2-Part e[8]	easter[1]
s0b44812	easter[8]	-18.5871	2-Part e[8]	easter[8]
s0b4481L	easter[8]	-19.5609	2-Part e[15]	easter[8]
s0b4481Y	easter[8]	-19.7912	2-Part e[15]	easter[8]
s0b45210	easter[8]	-22.3665	2-Part e[8]	easter[8]
s0b45212	easter[8]	-24.5982	2-Part e[8]	easter[8]
s0b45231	easter[15]	-7.7293	2-Part e[15]	easter[15]
s0b45291	easter[8]	-14.6856	2-Part e[8]	easter[8]
s0b45299	easter[15]	-22.0504	2-Part e[15]	easter[15]
s0b45310	easter[1]	-33.972	2-Part e[8]	easter[1]

Table 2: Choice of Current Easter Model (Data: Retails Sales Series 1992- 2006, Source: U.S. Census Bureau).

Series	Choice of Easter regressor	AICC Difference Best Linear vs. None	Choice of 2-Part Linear	Choice of 2-Part Linear vs. Linear
s0b44510	linear e[8]	-58.2565	2-Part linear e[15]	linear e[8]
s0b44520	linear e[15]	-95.7815	2-Part linear e[15]	linear e[15]
s0b44530	linear e[8]	-3.8603	2-Part linear e[8]	2-Part linear e[8]
s0b445X0	linear e[8]	-69.1474	2-Part linear e[15]	linear e[8]
s0b44600	linear e[15]	-19.8071	2-Part linear e[15]	linear e[15]
s0b44611	linear e[15]	-21.107	2-Part linear e[15]	linear e[15]
s0b44811	linear e[8]	-21.903	2-Part linear e[8]	2-Part linear e[8]
s0b44812	linear e[8]	-19.6466	2-Part linear e[8]	linear e[8]
s0b4481L	linear e[15]	-20.203	2-Part linear e[15]	linear e[15]
s0b4481Y	linear e[15]	-20.4691	2-Part linear e[15]	linear e[15]
s0b45210	linear e[15]	-22.0163	2-Part linear e[8]	2-Part linear e[8]
s0b45212	linear e[8]	-23.7728	2-Part linear e[8]	2-Part linear e[8]
s0b45231	linear e[15]	-7.4536	2-Part linear e[8]	linear e[15]
s0b45291	linear e[15]	-13.8148	2-Part linear e[8]	2-Part linear e[8]
s0b45299	linear e[15]	-20.0451	2-Part linear e[15]	2-Part linear e[15]
s0b45310	linear e[8]	-34.5496	2-Part linear e[8]	linear e[8]

Table 3: Choices of Linear and 2-Part Easter regression model (Data: Retails Sales Series 1992- 2006, Source: U.S. Census Bureau).

Series	Choice of Current vs. Linear	AICC Difference Current vs. Linear	Best Overall	AICC Difference Best vs. Easter[8]
s0b44510	linear e[8]	0.8109	linear e[8]	-0.8109
s0b44520	linear e[15]	0.2486	linear e[15]	-0.2486
s0b44530	easter[1]	-0.0363	easter[1]	-3.6136
s0b445X0	linear e[8]	1.4361	linear e[8]	-2.8613
s0b44600	linear e[15]	0.1148	linear e[15]	-0.1148
s0b44611	linear e[15]	0.2769	linear e[15]	-0.2769
s0b44811	easter[1]	-1.4287	easter[1]	-7.1796
s0b44812	linear e[8]	1.0595	Weekend-Weekday-Easter	-5.9601
s0b4481L	linear e[15]	0.6421	Weekend-Weekday	-2.8882
s0b4481Y	linear e[15]	0.6779	Weekend-Weekday	-3.0916
s0b45210	2-Part linear e[8]	0.8783	Weekend-Weekday	-15.2059
s0b45212	2-Part linear e[8]	0.629	Weekend-Weekday	-10.9756
s0b45231	easter[15]	-0.2757	Weekend-Weekday	-5.8173
s0b45291	easter[8]	-0.5907	Weekend-Weekday	-10.485
s0b45299	easter[15]	-1.2366	Weekend-Weekday	-11.1531
s0b45310	linear e[8]	0.5776	linear e[8]	-3.0754

Table 4: Choice of Easter Regressor (Data: Retails Sales Series 1992- 2006, Source: U.S. Census Bureau).

As is often the case with forecast error plots, the results are sometimes inconclusive, and there is no clear model choice indicated by the graphs. Often there is a split verdict, with a clear winner for lag one forecasts and another for lead 12 forecasts.

For the 16 Retail Sales series in this study,

- **6** had forecast error plots that showed significant forecast improvement using a model with some other Easter regressor than the `easter[8]` regressor;
- **7** had split preferences for the choice of model, and
- **3** were inconclusive.

For forecast error plots where the best model found by AICC was compared to using a model with no Easter regressor at all, the results differed greatly from the conclusions arrived at by AICC. Only six of the series with the AICC choice of Easter improve the forecast performance of the model over a model with no Easter. Curiously, most of these series are using either the `easter[1]` regressor or one of the linear Easter regressors, as is the case with Retail Grocery Sales (s0b44510) in Figure 3.

As before, often the plots were inconclusive or split in their verdict as to the best model. However, in five cases there appears to be no benefit to using Easter regressors in the regARIMA model, at least in terms of forecasting performance. Figure 4 shows an example for a series where the model without any Easter regressors was preferred, despite AICC consistently prefering the model with the Weekend-Weekday Easter regressors.

Further examination of these series should be done to determine if an Easter adjustment is suitable for these 5 series.

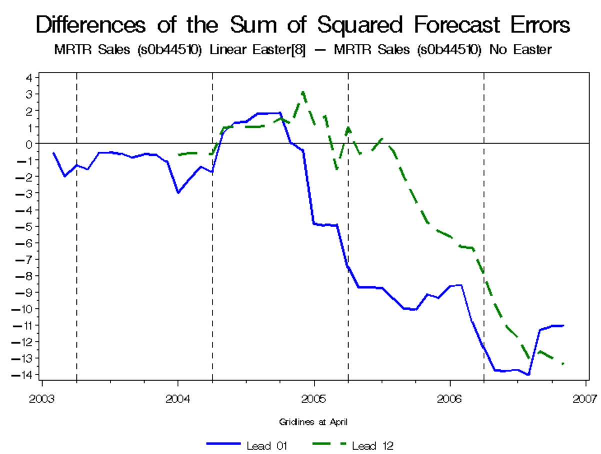

Figure 3: Forecast Error plot comparing forecasts from models with No Easter and Linear Easter[8] regressors for Retail Grocery Sales (s0b44510).

3 Constrained Stock Trading Day

Trading day regressors are often used in flow series such as retail sales to adjust for the effect that the different number of days in the calendar has on the level of the series. The regressors defined in Bell and Hillmer (1983) have been used in the X-12-ARIMA from its inception.

For inventory series, Bell (1984) and Bell (1995) developed regressors that could be used for the same purpose. For a given integer value w assumed to be the day of the month where inventory is taken, these regressors are defined as

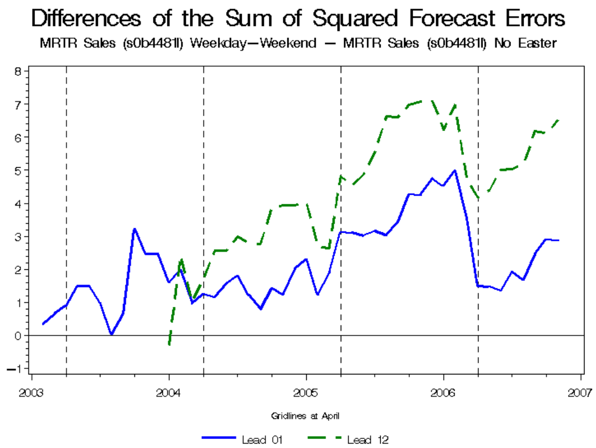

Figure 4: Forecast Error plot comparing forecasts from models with No Easter and Weekend-Weekday Easter regressors for Retail Sales of Miscellaneous Apparel and Luggage (s0b4491L) (source: U. S. Census Bureau).

$$I_{1,t} = \begin{cases} 1 & \tilde{w}^{\text{th}} \text{ day of month } t \text{ is a Monday} \\ -1 & \tilde{w}^{\text{th}} \text{ day of month } t \text{ is a Sunday} \\ 0 & \text{otherwise} \end{cases},$$
$$\cdots, \quad I_{6,t} = \begin{cases} 1 & \tilde{w}^{\text{th}} \text{ day of month } t \text{ is a Saturday} \\ -1 & \tilde{w}^{\text{th}} \text{ day of month } t \text{ is a Sunday} \\ 0 & \text{otherwise} \end{cases}, \tag{14}$$

where $\tilde{w}$ is the smaller of w and the length of month t; $w = 31$ assumes that inventory is taken at the end of the month.

Constrained flow trading day regressors were first developed for the TRAMO program (see Gómez and Maravall (1997)), and are also implemented in X-12-ARIMA as `td1coef` in the `variables` argument of the `regression` spec. The programs use a weekday-weekend contrast model that imposes separate equality constraints on weekday and weekend day of week coefficients, ie,

$$\begin{aligned} \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 \\ \beta_6 = \beta_7 \end{aligned} \tag{15}$$

where β_1 be the day-of-week coefficient for Monday, β_2 the day-of-week coefficient for Tuesday, etc.

The flow regressor that corresponds to this constraint is simple to generate. Let N_t^{WD} be the number of weekdays in month/quarter t and N_t^{SS} be the number of Saturdays and Sundays in month/quarter t. The constrained flow regressor implied by the constraint given in (15) is given below:

$$TD(t) = N_t^{WD} - \frac{5}{2}(N_t^{SS}) \tag{16}$$

Findley and Monsell (2007) derives how day-of-week effect constraints like the one used to generate the regressors in equation (16) can be imposed upon the stock trading day model of Bell (1984) and Bell (1995) used in X-12-ARIMA, and gives weights for deriving such a set of constrained trading day regressors from a stock trading day regression matrix with sample day w generated as in equation (14).

For this paper, we will generate a constrained stock trading day variable based on the flow trading day constraints given in (15). As shown in Findley and Monsell (2007), the stock trading day regressor that results from this constraint is

$$D_t = -\frac{3}{5}I_{1,t} - \frac{1}{5}I_{2,t} + \frac{1}{5}I_{3,t} + \frac{3}{5}I_{4,t} + I_{5,t} \tag{17}$$

where $I_{1,t}, I_{2,t}, ..., I_{5,t}$ are taken from the definition of stock trading day given in (14).

Trading day adjustments of inventory series are currently rare and are not done for production at the Census Bureau. One reason may be that the full stock trading day model is not parsimonious; there are too many parameters to estimate such a small effect. Perhaps using a constrained regressor will allow for an efficient estimation of the effect.

3.1 Analysis of Industrial Inventory Series

To examine the possibility that a constrained stock trading day effect can be found in Census Bureau inventory series, a group of 91 industrial inventory series will be modeled for stock trading day. We are going to assume that inventory is taken at the end of the month for these series, so we will be using end-of-month stock trading day regressors generated as in (14) with $w = 31$.

Three possible models are examined - a model without stock trading day regressors, a model with unconstrained end-of-month stock trading day, and a model with constrained end-of-month stock trading day as in (17).

The stock trading day regressors given above will be added to the model currently used in production at the Census Bureau. All the data sets are from January 1992 to October 2006.

To examine which inventory series would be most amenable to including the stock trading day, a standard log-likelihood difference asymptotic chi-square test will be used.

When one regARIMA model is of the correct type and is nested in (i.e. is a special case of) of another model, then for long enough time series, $-2\{L_N^{(1)} - L_N^{(2)}\} = 2\left\{L_N^{(2)} - L_N^{(1)}\right\}$ varies approximately like a chi-square variate with $n_p^{(2)} - n_p^{(1)}$ degrees of freedom. That is, asymptotically

$$-2\{L_N^{(1)} - L_N^{(2)}\} \sim \chi^2_{n_p^{(2)} - n_p^{(1)}} \tag{18}$$

holds, under standard assumptions, including the requirement that the true model is invertible, i.e. without unit magnitude roots in the MA polynomial (see Taniguchi and Kakizawa (2000), p. 61). The same result applies to AICC differences because $\left(n_p^{(1)} + 1\right)/N$ and $\left(n_p^{(2)} + 1\right)/N$ tend to zero as N increases.

Using a level of significance of $\alpha = 0.05$, there are 21 series where the model with no trading day was rejected in favor of a model with stock trading day. However, for one of the series, the trading day model was rejected for inducing a visually signficant trading day peak in the seasonally adjusted series (see Soukup and Findley (1999) for a definition of a visually significant spectral peak). Another series was rejected because the model without trading day had better forecasting performance than the model with stock trading day.

For the 19 remaining series:

- the unconstrained stock trading day regressors were significant for three series;
- the constrained stock trading day regressor was significant for eight series;
- both the constrained and unconstrained regressors were significant for eight series.

When both the constrained and unconstrained regressors were found to be significant, the constrained stock trading day was always preferred over the unconstrained stock trading day regressors using an appropriate chi-square test. However, the unconstrained stock trading day regressors were preferred for two of these series when further examination showed that the constrained stock trading day left a visually significant visual peak in the regARIMA residuals.

To summarize: in 14 of the 19 series where a stock trading day regressor was found significant, the constrained stock trading day model was the preferred model. In addition, for 12 of the 14 series where the constrained stock trading day model is preferred, additional model criteria were found to support this choice, including

- reducing visually significant spectral peaks;
- reducing the number of lags with significant Ljung-Box statistics;
- reducing out-of-sample forecast errors for one and 12 step ahead forecasts.

4 Further Research

For the Easter regressors, work is needed to understand more fully the lack of agreement between the AICC results and the graphs of the evolving forecast error. Examining model coefficients generated at each stage of the history run may provide insight on why the forecast performance seems so erratic when only the Easter variables are changed. Applying the findings of Wills (2006) to determine if many of the series would benefit from Easter adjustment will also be done.

Also, more work can be done refining the Weekend-Weekday Easter model. Giving the different weekend dates different weights (i.e., giving more weight to Saturdays and less weight for Fridays and Sundays) should be examined.

For the constrained stock trading day regressors, the regressors will be evaluated for retail inventories to see if more series with strong evidence of stock trading day can be found. The current results are promising enough to have this regressor included in a future release of X-12-ARIMA.

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