RESEARCH REPORT SERIES
(Statistics #2018-01)

Modeling of Holiday Effects and Seasonality in Daily Time Series

Tucker S. McElroy
Brian C. Monsell
Rebecca J. Hutchinson

Center for Statistical Research & Methodology
Research and Methodology Directorate
U.S. Census Bureau
Washington, D.C. 20233

Report Issued: January 23, 2018

Modeling of Holiday Effects and Seasonality in Daily Time Series

Tucker S. McElroy[*,†] Brian C. Monsell[‡,†] and Rebecca J. Hutchinson[§]

January 23, 2018

Abstract

This paper provides analyses of daily retail data, extracting annual and weekly seasonal patterns along with moving holiday effects, using an unobserved components framework. It is shown that the weekly seasonality, which corresponds to the trading day effect observed in monthly time series, can be treated in a dynamic framework via stochastic unobserved component models. A secondary result is the measurement of economically significant holiday effects in retail sector data, where the impact of Black Friday, Cyber Monday, Easter, Superbowl Sunday, and Labor Day is explicitly determined.

Keywords: Big Data; Seasonal Adjustment; Signal Extraction

1 Introduction

Statistical agencies around the world are under increasing pressure from the public to generate more data: more stratifications, finer regional detail, and greater frequency of observation. In the post-war era, central banks and data publishing agencies in the developed world have typically published economic data at a national level at quarterly or monthly frequency, with some sub-national estimates available in particular cases. However, in the last decade – with the advent of new information technologies, and the concomitant impact upon democracies of the surge in data – modern cultures have been transformed by the wealth of measured phenomena, and the citizen craves ever more information. As public institutions that serve the tax-payer, statistical agencies in democracies are being compelled to offer more; failure to meet this demand ensures that the public will resort to less reputable vendors of data.

[*]Center for Statistical Research and Methodology, U.S. Census Bureau, 4600 Silver Hill Road, Washington, D.C. 20233-9100, tucker.s.mcelroy@census.gov

[‡]Center for Statistical Research and Methodology, U.S. Census Bureau, 4600 Silver Hill Road, Washington, D.C. 20233-9100, brian.c.monsell@census.gov

[§]Economic Directorate, U.S. Census Bureau, 4600 Silver Hill Road, Washington, D.C. 20233-9100, rebecca.j.hutchinson@census.gov

The U.S. Census Bureau (USCB) has for many decades published monthly retail and construction data at a national level, based on a survey of businesses. Given that this data – in its seasonally adjusted form – is heavily used by economists, politicians, and journalists as the basis for forming opinions about the economy, there is frequent demand for weekly or daily time series, available in fine disaggregations (of business type) over a state or county regional map. There is some precedent towards moving in this direction: the Bureau of Labor Statistics has long published its own employment numbers on a weekly national basis (Pierce, Grupe, and Cleveland (1984)). Also, the Longitudinal Employment and Household Dynamics database of the USCB currently publishes data on hires and separations county by county, though on a quarterly schedule (Abowd and Vilhuber, 2011). As an experiment in Big Data, USCB participated in a pilot project along with the Bureau of Economic Analysis, Palantir Technologies, Inc., and FirstData. FirstData furnished credit card transaction data via a Palantir designed aggregation tool from which daily regional time series can be tabulated.

FirstData is the largest credit card payment processor in the United States of America. All of the credit, debit, prepaid, and EBT (Electronic Benefit Transfer) transactions for each merchant that utilizes the FirstData service are recorded, with information on authorizations, settlements, and an exact time stamp. These items, from over 600 merchant categories, are then aggregated into the NAICS (North American Industry Classification Systemt) codes at a daily frequency, with adjustments for the local time zone of the merchant. The result is a "big data" retail database, covering all types of cards, all banks, all networks, and all fifty states, as well as all customer segments and all sizes of merchants.

In order to gauge the utility of this high frequency retail data, several technical questions must be addressed. Firstly, are the daily retail series from FirstData suitable proxies – once aggregated to a monthly level – of the corresponding USCB monthly retail series? Secondly, to what extent do analyses of the daily data – the identification of trend and seasonality, as well as moving holiday effects – have ramifications for the seasonal adjustment of known monthly series? I.e., can daily data provide greater insight and superior seasonal adjustment of the monthly data? Thirdly, can daily data from FirstData be utilized to assist in the publication of new, more timely USCB retail series? This paper is primarily focused on the second question, leaving the first and third topics for future work.

Daily time series represent a substantial challenge for conventional time series modeling methodologies, which were developed to analyze monthly and quarterly data. (Harvey, Koopman, and Riani (1997) and Cleveland and Scott (2007) treat the weekly case.) One key challenge is the presence of multiple types of seasonality, having weekly, monthly, or annual periods; see, for example, Weinberg et al. (2007) and Hyndman and Fan (2010). Another key challenge is that trend and annual seasonality (i.e., seasonal effects with an annual period) are difficult to distinguish, even with a fairly large sample. However, there are enormous potential benefits to statistical agencies of utilizing daily data, with respect to the second question raised in the previous paragraph. The additional information furnished could assist with a proper understanding of trading day effects in monthly data, where the actual dynamic is essentially masked and distorted by the monthly sampling scheme, and the vagaries of the Gregorian calendar (Bell and Hillmer, 1983). Secondly, the analysis of daily data permits a direct engagement with moving holiday effects, which – like trading day – are essentially corrupted by monthly sampling; see Findley, Wills, and Monsell (2005) and Roberts et al. (2010).

There are comparatively few papers addressing signal extraction for high frequency time series. One ground-breaking paper is De Livera, Hyndman, and Snyder (2011), which proposes stochastic unobserved component models with a trigonometric form. These models are essentially the same as the stochastic cycle models of Harvey (1989) and Harvey and Trimbur (2003), although adapted to handle specific frequencies, and with a single source of error, i.e., all the latent components are driven by a single innovation sequence. This single source of error approach represents a substantial departure from the broad consensus on signal extraction, and appears to be motivated by computational considerations. The framework developed in McElroy and Trimbur (2015) and McElroy (2017) is similar in spirit, but with the key difference that each latent component is driven by innovations that are uncorrelated with one another. This is more consistent with prevailing practice in econometrics, although the question of cross-correlation in latent innovations is ultimately an empirical one (cf. discussion in McElroy and Maravall (2014)).

This paper studies daily retail data through the tools of univariate unobserved component models. We show the efficacy of these models for capturing dynamics that are latent within the daily series, and how these extracted dynamics are related to the dynamics of a monthly time series. Various important moving holiday effects are modeled and assessed. The key novel findings of the paper are that weekly seasonality – corresponds to monthly trading day phenemona once the daily data are aggregated to a monthly frequency – can be effectively modeled and extracted using this paper's statistical methods; secondly, that annual seasonality (which corresponds to the usual seasonality of monthly and quarterly time series) can be captured, and it exhibits the essential features seen in monthly seasonal factors, albeit at a daily time interval; thirdly, short windows for moving holiday effects are quite effective in daily time series, and their impact is not only statistically significant, but quite obviously economically significant as well. We present the data in Section 2, discuss our modeling methodology in Section 3, the applications to signal extraction and algorithmic innovations in Section 4, and the empirical results in Section 5.

2 Daily Retail Series from FirstData

We focus our study on nine daily retail series collected by FirstData, whose attributes are summarized in Table 1. Each time series is daily, covering the period from October 1, 2012 through April

12, 2016. In order to avoid disclosure, each series has first been divided by its October 1, 2012 value, so that the beginning of each series is at unity. As the nine series each correspond to different facets of the retail economy, it is to be expected that certain holiday features may be present to a greater or lesser degree depending on the sector. For example, we might suppose a Super Bowl Sunday effect to be relevant for Sporting Goods Stores, and an Easter effect for Shoe Stores.

Plots of the time series are given in Figure 1. There is a complex seasonal pattern present, along with a weak trend; in fact, the seasonality can be decomposed into a primary annual pattern and a secondary weekly pattern – this is easiest to see in series 44511. While a monthly pattern seems plausible, it is not a strong effect in these nine series, and can be omitted with no loss to model quality. Some of the series (44311, 4482, and 45111) have salient additive outliers, although in most cases these aberrations are actually the result of moving holiday effects, such as Easter, Black Friday, Labor Day, and Cyber Monday.

Label	Title	Epithet
44311	Appliance, Television, and Other Electronics	Electronic
44411	Home Centers	Home
44511	Supermarkets and Other Grocery Stores	Grocery
44814	Family Clothing Stores	Clothing
4482	Shoe Stores	Shoe
45111	Sporting Goods Stores	Sport
45211	Department Stores	Department
45291	Warehouse Clubs and Superstores	Warehouse
45411	Electronic Shopping and Mail Order	Mail

Table 1: Label, Title, and Epithet for daily retail data.

In order to get a crude initial analysis of the main features of each series, we compute spectral density estimates via an autoregressive spectral estimator (Tiao and Tsay, 1983) and mark with vertical lines the chief frequencies of potential interest (Bell and Hillmer, 1983). In Figure 2 we display the spectral densities in units of a daily frequency; spectral peaks of a cusp-like convexity are known to correspond to periodic behavior in the process' autocorrelation function (Findley, 2005), where the period is given by 2π divided by the radians frequency of the peak. In the spectral plot, the x-axis has already been normalized, so that the ordinates correspond to the number of cycles per year (i.e., the number of times a phenomenon occurs per year). The peaks indicated by the vertical red lines correspond to $365/7$, $2 \cdot 365/7$, and $3 \cdot 365/7$, or roughly 52, 104, and 156 – hence once, twice, and thrice a week.

The green line denotes twelve cycles per year, i.e., the monthly frequency, and no such effect is apparent in the series. The annual cycle, which corresponds to more conventional notions of

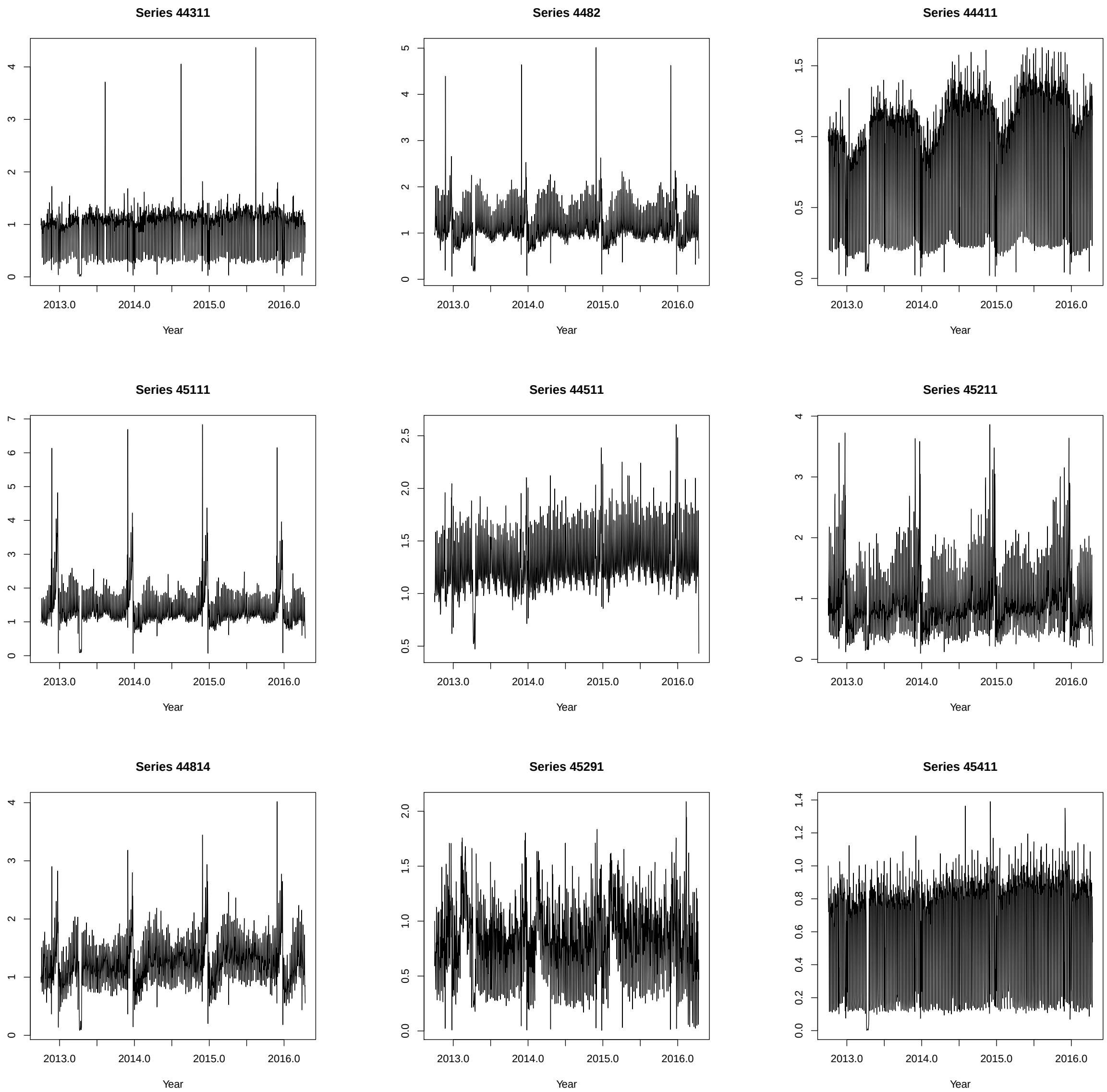


Figure 1: Time series plots of daily retail data. Each series covers the period from October 1, 2012 through April 12, 2016.

seasonality, is the blue line. One of the key challenges with daily data is the entanglement of trend and annual seasonality. This is because the annual frequency is $2\pi/365$, or .99 degrees, which is very close to zero, the trend frequency – this poses a difficulty for likelihood evaluation and signal extraction computation. However, for these nine series the change in level – due to the limited number of years for which there is measurement – is quite limited, which indicates the adequacy of very simple trend models. These empirical considerations guide our choice of structural models, discussed in the next section.

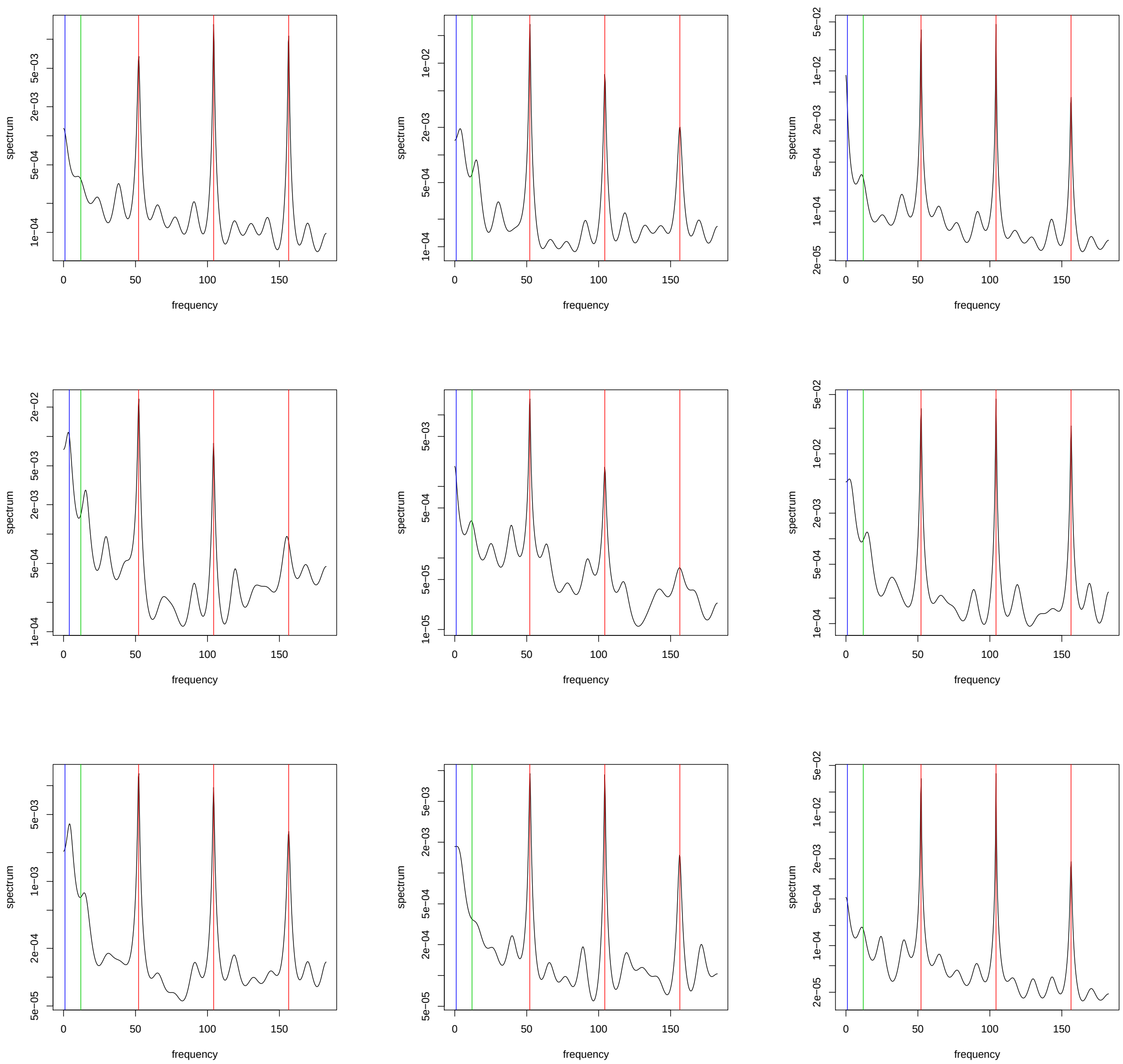


Figure 2: AR spectral density estimate of daily retail data. Vertical red lines correspond to once a week, twice a week, and thrice a week phenomena; the blue line corresponds to annual phenomena, and the green line corresponds to monthly phenomena.

3 Modeling

Our basic model begins with the daily data, not paying particular attention to the day of week pertaining to each time index – this will be captured through the weekly seasonal effect. We propose a model for each univariate series $\{y_t\}$, which has been appropriately transformed, providing an additive decomposition into latent components:

$$y_t = \mu_t + \xi_t^a + \xi_t^w + \iota_t + z_t. \tag{1}$$

Here $\{\mu_t\}$ is a stochastic trend, $\{\iota_t\}$ is a stationary transient (or irregular), and the seasonals come in weekly ($\{\xi_t^w\}$) and annual ($\{\xi_t^a\}$) varieties. We have omitted a monthly seasonal component for the retail data, as their inclusion was not really warranted by the spectral analysis. Fixed effects are incorporated through z_t, which consists of a set of regressors (e.g., moving holidays and outliers) – these may differ from series to series. Each of these latent processes involves model parameters, which will need to be estimated in a preliminary modeling stage.

Treating these estimated parameters as if they are known, we may proceed to signal extraction, which will allow optimal estimation of trend, seasonal, fixed, or transient effects. The trend and seasonal latent processes will be supposed to be non-stationary, being defined such that differencing by a polynomial $\delta^\omega(B)$ yields a mean zero, stationary time series. Here, for any $\omega \in [0, \pi]$, we define

$$\delta^\omega(B) = 1 - 2\cos(\omega)B + B^2.$$

As a special case $\delta^0(B) = (1 - B)^2$. In general, the null space of $\delta^\omega(B)$ has the basis of time series $e^{\pm i\omega t}$, or all deterministic time series of frequency ω. Now the frequencies appropriate for annual and weekly effects in a daily time series are simply given by dividing the corresponding daily period into 2π, yielding $2\pi/365$ and $2\pi/7$. (If desired, leap year could be accounted for by taking the average year length over four years to be 365.25, but such subtleties make no difference to model fitting.) These are the chief harmonics, although higher multiples of such frequencies might also be considered. Combining the weekly frequency with its double and triple frequency harmonics, we obtain

$$\delta^{2\pi/7}(B) \cdot \delta^{4\pi/7}(B) \cdot \delta^{6\pi/7}(B) = 1 + B + B^2 + B^3 + B^4 + B^5 + B^6 =: U^w(B),$$

which is verified by polynomial arithmetic. We use each of these differencing polynomials to define latent components by imposing that the differenced component is a white noise process. In each case we denote the differenced component by putting a ∂ symbol in front of it. Therefore we have

$$\begin{aligned} \partial\mu_t &= \delta^0(B)\mu_t \sim \mathrm{WN}(0, \sigma_\mu^2) \\ \partial\xi_t^a &= \delta^{2\pi/365}(B)\xi_t^a \sim \mathrm{WN}(0, \sigma_a^2) \\ \partial\xi_t^w &= U^w(B)\xi_t^w \sim \mathrm{WN}(0, \sigma_w^2) \\ \iota_t &\sim \mathrm{WN}(0, \sigma_\iota^2). \end{aligned}$$

The innovation variances are unknown parameters – in this case there are four of them. For the retail series there are 1290 observations, so the model is quite parsimonious. When there is very little change to the trend the trend innovation variance will be approximately zero, indicating a fixed linear trend (i.e., $\mu_t = \eta_1 + \eta_2\, t$ for parameters η_1, η_2). In our modeling, because the series length is short, we found that a constant trend (setting $\eta_2 = 0$) gave adequate results. For longer series with more nuanced trending behavior such a crude device would be undesirable[1].

The spectral plots in Figure 2 indicate that the three weekly peaks can have varying width; as argued in McElroy (2017), it is advantageous to entertain a more nuanced weekly specification, known as the atomic specification:

$$
\begin{aligned}
\xi_t^w &= \xi_t^{(1)} + \xi_t^{(2)} + \xi_t^{(3)} \\
\partial \xi_t^{(1)} &= \delta^{2\pi/7}(B)\xi_t^{(1)} \sim \mathrm{WN}(0, \sigma_1^2) \\
\partial \xi_t^{(2)} &= \delta^{4\pi/7}(B)\xi_t^{(2)} \sim \mathrm{WN}(0, \sigma_2^2) \\
\partial \xi_t^{(3)} &= \delta^{6\pi/7}(B)\xi_t^{(3)} \sim \mathrm{WN}(0, \sigma_3^2).
\end{aligned}
$$

This atomic specification yields five variance parameters for the full model. The fixed effects z_t take the form

$$z_t = x_t' \beta.$$

Here x_t is a vector of r regressors, and β is the corresponding parameter. These regressors include the fixed effects, namely additive outliers (an indicator regressor) and moving holiday effects, such as Easter, Labor Day, Cyber Monday, Super Bowl Sunday, and Black Friday (we also considered Chinese New Year, but this holiday had no impact upon any of the retail series).

Holiday regressors are constructed by first determining the calendar dates for the actual holiday, say with day index t_*, and declaring a window of times $[t_* - b, t_* + f]$ for which the activity is increased or decreased. The initial regressor is just the indicator on the window, taking value one there and zero at other times; this window is present for every calendar year. Also, note that while typically $b, f \geq 0$, these can be negative as well. For example, a post-holiday effect is measured by taking $b < 0$ and $f > 0$.

From these initial regressors, we compute the mean over all calendar years (as long as possible), and subtract this long-term mean to get the holiday regressor. See Findley and Monsell (2009) for background discussion. If multiple holiday windows are desired, several such regressors can be added to the model. For example, an Easter-day effect utilizes $b = f = 0$, whereas a pre-Easter effect is achieved via $b = 8$, $f = -1$, and both regressors can be inserted in the model.

As regards trading day effects, these phenomena will actually be measured through the weekly seasonal component, and so no regressor is needed, unlike the case of monthly retail time series. (Put another way, indicator regressors for day-of-week, once de-meaned to render them orthogonal to the long-term mean, will be in the null space of $U^w(B)$, and hence are not identifiable.) The fixed trend effects are labeled under the μ_t process instead of z_t; we have $\mu_t = \eta_1$.

In order to fit these latent component models, it is necessary to obtain the reduced form representation for the observed process. Since all the differencing operators are distinct (i.e., the

[1]Modeling with a stochastic trend of order one or two resulted in failed convergence of the likelihood optimization, with the algorithm terminating at a saddlepoint, indicating the possibility of over-differencing.

polynomials share no common roots), the minimum differencing polynomial that reduces the data to stationarity is given by their product, i.e.,

$$\delta(B) = \delta^{2\pi/365}(B) \cdot U^w(B) = \delta^{2\pi/365}(B) \cdot \delta^{2\pi/7}(B) \cdot \delta^{4\pi/7}(B) \cdot \delta^{6\pi/7}(B).$$

Applying $\delta(B)$ to (1) yields

$$\begin{aligned} \partial y_t = \delta(B) y_t &= \delta^{2\pi/365}(B) \cdot U^w(B) \mu_t \\ &+ U^w(B)\, \partial \xi_t^a \\ &+ \delta^{2\pi/365}(B)\, \partial \xi_t^w \\ &+ \delta^{2\pi/365}(B) \cdot U^w(B)\, \iota_t \\ &+ \delta^{2\pi/365}(B) \cdot U^w(B)\, z_t. \end{aligned}$$

In the case of atomic weekly seasonals, we also have the expression

$$\partial \xi_t^w = U^w(B) \xi_t^w = \delta^{4\pi/7}(B) \cdot \delta^{6\pi/7}(B)\, \partial \xi_t^{(1)} + \delta^{2\pi/7}(B) \cdot \delta^{6\pi/7}(B)\, \partial \xi_t^{(2)} + \delta^{2\pi/7}(B) \cdot \delta^{4\pi/7}(B)\, \partial \xi_t^{(3)}.$$

The autocovariance sequence of $\{\partial y_t\}$ is easily computed from these equations. In fact, the equation for ∂y_t takes the form of three (five in the case of atomic weekly seasonals) independent vector moving average processes, of various orders, whose moving average polynomials are given by the various products of differencing polynomials, each being driven by independendent white noises of variances given above.

Regarding the first and last terms, the fixed effects: we simply apply the differencing operator to each component regressor of μ_t and x_t – so long as this is not annihilated, the fixed effect is identifiable (otherwise, it is redundant with at least one of the components, and can safely be eliminated from the model). Let us denote this vector of differenced regressors, for the trend and non-trend effects, via

$$\partial m_t = \delta(B) m_t \qquad \partial x_t = \delta(B)\, x_t.$$

Then $\partial y_t - \partial m_t - \partial x_t' \beta$ is mean zero, with autocovariance structure given by summing the autocovariances of the three (or five) latent moving average processes. Thus, the parameter vector is $\theta' = [\eta_1, \beta', \sigma_a^2, \sigma_w^2, \sigma_\iota^2]$ (in the case of atomic weekly seasonals, we replace σ_w^2 by $\sigma_1^2, \sigma_2^2, \sigma_3^2$), which in the *sigex* software[2] could be estimated via either an OLS and MOM (method of moments) scheme, or preferably via maximizing the Gaussian likelihood. The estimates will be denoted $\widehat{\theta}$.

With a fitted model in hand, we can proceed to check the goodness-of-fit via examination of the time series residuals. If the model seems to be adequate, we may consider more parsimonious nested alternative models. For example, if σ_w^2 gets estimated as close to zero, we might attempt

[2] *sigex* is a suite of R routines that allow modeling, forecasting, and signal extraction for multivariate time series, available by request from the first author.

to fit a nested model wherein the weekly seasonal component is removed. However, we should still insert two regressors of the form $\cos(\omega t)$ and $\sin(\omega t)$ with $\omega = 2\pi/7$, which are both in the null space of the operator $\delta^{2\pi/7}(B)$. If this substitution is warranted – assessed via an AIC comparison – then we substitute a fixed effect (handled in the mean of the time series) for the stochastic effect (handled in the variance of the time series), corresponding to a more stable phenomenon. This is like having a spectral peak that is infinitely high and skinny – in contrast, a cusp-like peak with a broad base and finite height corresponds to a dynamic phenomenon, that substantially differs from exactly periodic behavior over a longer time span (Soukup and Findley, 2000). If the regression parameter is negligible as well, then the component could be completely eliminated.

4 Signal Extraction

Once the modeling is complete, we can proceed towards signal extraction, which is concerned with extracting (estimating) the components of interest. Actually, in the case of fixed components there is nothing further to do, because we merely take

$$\widehat{\mu}_t = \widehat{\eta}_1 \qquad \widehat{z}_t = \underline{x}_t' \, \underline{\widehat{\beta}}.$$

The stochastic components are more subtle to estimate. The seasonal components are centered about zero, having expectation zero (conditional on their initial values). However, the seasonally adjusted component consists of $\mu_t + \iota_t$, and should be centered around the overall level of the series. For this reason, we should not enforce $\eta_1 = 0$ even when its estimate has an insignificant t-statistic, because doing so would generate a seasonal adjustment centered about zero, which might be offset from the true level of the data.

Our approach reflects a frequentist philosophy; a Bayesian analyst would weave model selection, parameter estimation, and signal extraction into a seamless garment. Although arguably less elegant, the frequentist approach is computationally simpler and still provides correct and relevant results. Formulas for signal extraction are discussed in McElroy (2008); these formulas are implemented in *sigex*. Alternatively, one could embed the model into a state space formulation, and utilize the Kalman filter to evaluate the Gaussian likelihood (*sigex* uses the efficient Durbin-Levinson algorithm instead), followed by a state space smoother to get signal extraction results. The pros and cons of these approaches are discussed in McElroy and Trimbur (2015) and McElroy (2017).

As compared with monthly time series, which has a single (annual) seasonal component, the daily time series has two types of seasonality. One can always obtain derived monthly series by aggregation (assuming a flow structure) of the daily series, and this operation would exactly annihilate any monthly seasonality. The annual seasonal would be left, corresponding to the regular seasonality observed in monthly data, and the weekly seasonal would actually become the trading day component. To see why this is so, consider the latent $\{\xi_t^w\}$, whose extraction could actually be plotted as seven sub-series, corresponding to each day of the week. Such an exercise is often done for monthly time series, by viewing the extracted seasonal as twelve monthly sub-series of seasonal factors. It may be that one of the daily sub-series (e.g., Saturday) is quite a bit higher than the others – this will contribute more highly to a given month than another sub-series (e.g., Tuesday) for which little activity occurs. (This would be especially relevant for retail data.) If a given month includes five Saturdays rather than four, its monthly aggregate will be higher than is typical, due to the calendrical composition – this is known as the trading day phenomenon (Findley, 2005).

In fact, when $\sigma_w^2 = 0$ and the weekly seasonal reduces to a fixed effect, we obtain a single cosine and sine (of period seven days) fitted to the weekly seasonality, which is assumed to recur with exact repetition through the sample. We obtain added flexibility with the atomic weekly seasonal model: if all of these are fixed effects (each having variance zero), then the weekly seasonal essentially becomes the sum of six regression effects, corresponding to phenomena occuring once a week, twice a week, and thrice a week. While these regressors will not exactly correspond to daily trading regressors (which would just be an indicator for each day of week, for a total of six indicators), they form an approximation. In order to get a more nuanced description of trading day, and the weekly seasonality, one might consider modeling the daily data as a weekly seven-variate time series – but this generates substantial modeling and computational challenges, where the advantages are unclear.

These arguments indicate that the weekly seasonal effect with $\sigma_w^2 = 0$ corresponds to trading day, whereas letting $\sigma_w^2 > 0$ allows for a stochastic trading day effect in monthly data; alternative approaches to the modeling of dynamic trading day are discussed in Bell (2004) and Maravall and Pérez (2012). On the other hand, the monthly and annual seasonal effects correspond to more conventional notions of seasonality. Again, any monthly effects are never observed in monthly time series due to aliasing; these might correspond to utility usage and payroll time series, or any kind of economic activity conducted on a monthly basis. Seasonal adjustment entails the removal of all seasonal components, along with moving holidays as judged appropriate. (Some holidays exhibit calendrical oscillations that accord with seasonal frequencies, while others have longer periods that indicate they should be associated with the transient irregular.)

5 Results

5.1 Holiday Specifications

For each series, an initial model specification was utilized with six moving holiday effects (Easter, Black Friday, Cyber Monday, Labor Day, Super Bowl Sunday, and Chinese New Year) and a fixed linear trend. A summary of the pertinent holiday effects is given in Table 2; the significant t-statistics for these holiday effects are given in Table 3. For each series, holiday effects with insignificant t statistics (at the 5% level) were removed, along with the trend slope if appropriate (but the trend level was always retained, even if insignificant), and the model refitted. In each case, such omissions of insignificant holiday effects generated an AIC improvement of two to six points. On a computer (Intel Core 2.80 GHz with 8 GB RAM) the maximum likelihood optimization took under two minutes of time, and the signal extraction results required about seven minutes; this is negligible in comparison to the amount of time spent modeling.

Epithet	Label	Holiday Effects
Electronic	44311	**pre-Easter**, *Easter-day*, *LD*, **BF**, **CM**
Home	44411	*Easter-day*, *LD*, **CM**
Grocery	44511	*pre-Easter*, **LD**, *BF*
Clothing	44814	*Easter-day*, **pre-Easter**, *LD*, **BF**
Shoe	4482	*Easter-day*, **pre-Easter**, **LD**, **BF**
Sport	45111	*SBS*, *Easter-day*, **BF**, *CM*
Department	45211	*Easter-day*, **BF**
Warehouse	45291	*Easter-day*, *LD*, **CM**
Mail	45411	**pre-Easter**, *LD*, *BF*, **CM**

Table 2: FirstData daily retail series studied, with identified holiday effects (bold for positive, italics for negative).

Epithet Label	Super Bowl	Easter Day	Labor Day	Black Friday	Cyber Monday	
Electronic	44311		-3.51	-7.11	8.51	3.32
Home	44411		-1.94	-10.63		2.89
Grocery	44511			2.41	-2.99	
Clothing	44814		-6.92	-1.72	20.04	
Shoe	4482		-10.80	2.90	39.79	
Sport	45111	-1.69	-5.47		34.21	3.27
Department	45211		-2.12		15.52	
Warehouse	45291		-2.17	-5.38		2.40
Mail	45411			-9.26	-4.96	5.67

Table 3: FirstData daily retail series studied, with t-statistics for significant holiday effects.

The time series residuals each indicate some residual autocorrelation was present, so additional refining of the basic model is possible. However, our objective is to extract the annual and weekly seasonality, and the postulated models are sufficient for this purpose; the seasonal adjustments were in each case adequate, indicating that the extraction of annual and weekly seasonality was successful. The spectral density plots of the seasonally adjusted component are given in Figure 3, with vertical lines having the same definitions as in Figure 2. The spectral estimates (not displayed) for the annual and atomic weekly components have a single pole form, confirming the efficacy of the extraction.

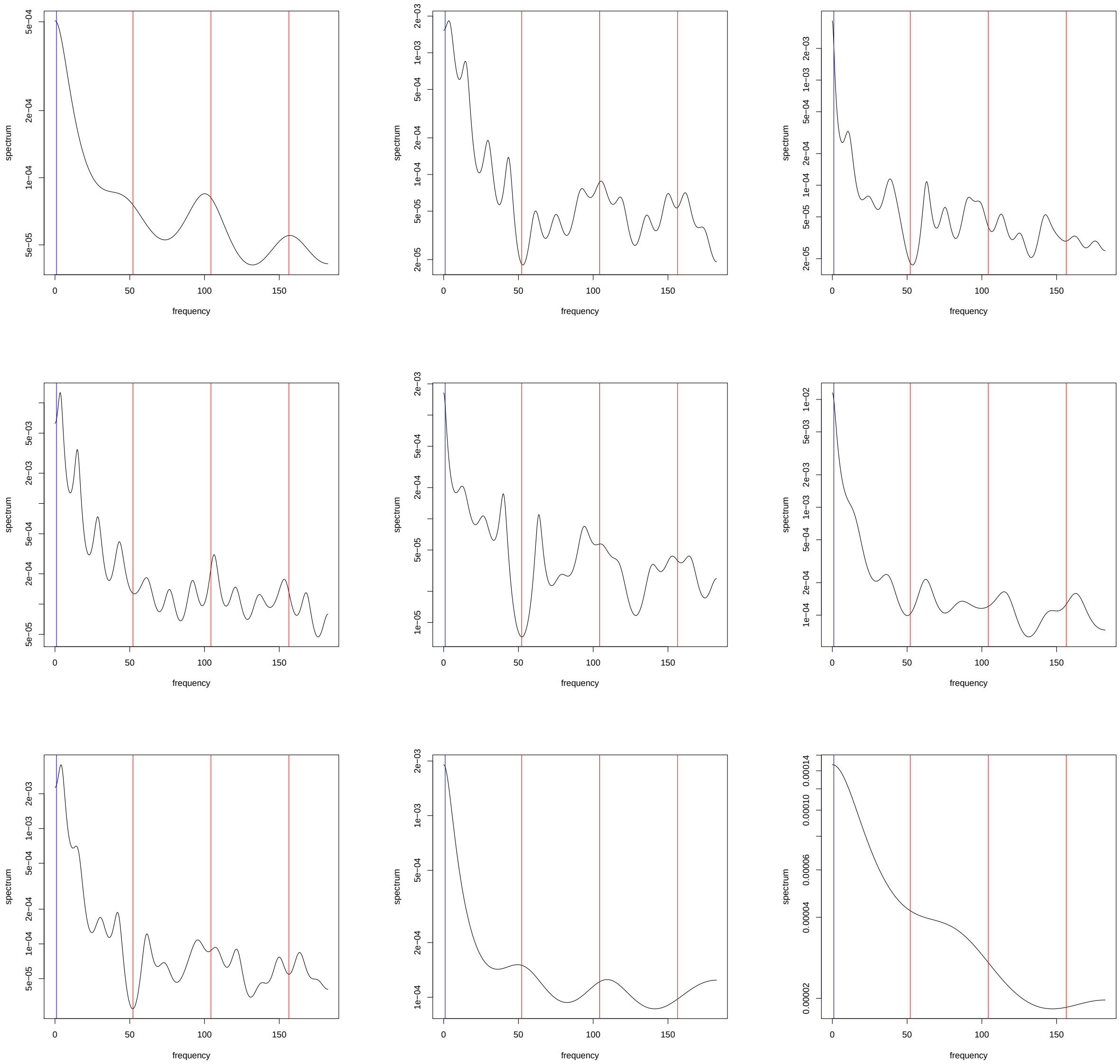


Figure 3: AR spectral density estimate of seasonally adjusted daily retail data. Vertical red lines correspond to once a week, twice a week, and thrice a week phenomena; the blue line corresponds to annual phenomena.

5.2 Individual Holidays

Below there are brief summaries of the results for the holidays shown in Table 3.

5.2.1 Super Bowl Sunday

NAICS Code 45111 (Sporting Good Stores) is the only series with a significant Super Bowl Sunday Effect. We often observe an increase in television sale promotion in the days and weeks leading up to Super Bowl Sunday. The daily data, however, do not show this effect to be present in NAICS Code 44311, which captures data from appliance, television, and other electronic stores. Additionally, Super Bowl Sunday is often considered a food holiday; the lack of a significant effect in the supermarket and grocery stores (NAICS Code 44511) is somewhat surprising. It is possible that this food shopping is done in the days leading up to Super Bowl Sunday rather than on Sunday itself.

5.2.2 Easter Day

Given that many retailers have no or reduced shopping hours on Easter Sunday, it was not surprising that seven of the eight low aggregation series had a negative and significant Easter day holiday effects.

5.2.3 Labor Day

Labor Day is another holiday where, while retailers may remain open for business, they may have reduced business hours. Thus the significant, negative effects are not surprising in so many of the series. The significant positive effect for supermarket and grocery stores (NAICS Code 44511) can possibly be explained by a shifting of shopping typically done on Saturday and Sunday to the Monday of the Labor Day three-day weekend. The positive and significant Labor Day effect for Shoes stores (NAICS Code 4482) is less intuitive, as we would have expected it to behave like other non-grocery stores.

5.2.4 Black Friday

Black Friday is likely the most famous shopping holiday at this time, though Cyber Monday is a close rival. Thus it is not surprising that many of our retail series demonstrate a positive and significant effect on a day known for deep discounts and limited time deals. Additional, the negative and significant effect in the supermarket and grocery store series makes sense, as sales in those stores typically fall on the days before Thanksgiving when shoppers prepare for Thanksgiving dinner.

5.2.5 Cyber Monday

Cyber Monday is a holiday centered upon online shopping. The positive and significant effects in the lower aggregate NAICS code series may be attributed to online sales captured by FirstData being featured in those NAICS codes rather than in the Non-Store Retailer NAICS Code (45411).

At the time of the pilot, FirstData and Palantir were working on a better way to identify and classify e-commerce transactions.

5.2.6 Non-store Retailers

NAICS Code 45411 captures those non-store retailers whose business is not necessarily limited or constrained to normal retail hours. These retailers can include online, mail-order, or catalog retailers. Of particular interest for this series are the shopping holidays that follow Thanksgiving. Black Friday typically has a low aggregate focus, while Cyber Monday has an e-commerce focus. We observe this in the series' results with the negative and significant Black Friday, as well as the positive and significant Cyber Monday results.

5.3 Signal Extraction Results

Figure 4 displays the signal extraction results for Appliance Stores (44311). There are two panels to this plot (and others referenced in this section). The upper panel shows the data in black, the seasonal adjustment in blue, the annual component in olive, and the weekly component in purple. All components are shaded, with the width of shading corresponding to twice the square root of the signal extraction mean squared error. The lower panel shows the data in black, the atomic weekly components in purple, and the holiday effects (pre-Easter and Easter Sunday, Cyber Monday, Black Friday and Labor Day) in green.

For four of the remaining series – Home Center Stores (44411, see Figure 5), Department Stores (45211, see Figure 6), Warehouse Stores (45291, see Figure 7), and Electronic Shopping and Mail Order (45411, see Figure 8) – there seemed to be some overall movement to the level, and it was statistically plausible that $\eta \neq 0$. However, due to the short length of the series we deemed that putting a drift in the trend might produce unwarranted conclusions about the longer-term behavior in the series, and therefore we have chosen to present the results with the more conservative choice, namely with $\eta = 0$. As for the other four series – Supermarket Stores (44511, see Figure 9), Clothing Stores (44814, see Figure 10), Shoe Stores (4482, see Figure 11), and Sporting Goods Stores (45111, see Figure 12) – we could not reject the null hypothesis that $\eta = 0$. Hence for all nine series, the seasonal adjustment (blue) consists of a level-shifted irregular component.

Several features are apparent from these extractions. The annual component (olive) in some cases contains monthly oscillations. Note that the extracted component is centered around zero, but has been shifted downwards in the figures so as to facilitate visualization. The weekly component in each series has no annual oscillation, although in some cases the amplitude appears to have an annual period. Neither is there any weekly seasonality in the annual component, which is verified by spectral peak plots (Figure 3).

Some of the anomalies in the data are captured in the annual component – e.g., a dip apparent in Spring 2013 for many of the series, which was explained (by Palantir) as a weakness in the collection of merchant data at that time. Other aberrations are assigned to the irregular, and hence are featured in the seasonal adjustment (blue). We also note that the weekly component (purple) is obviously dynamic, which indicates that the use of deterministic regressors, as would occur in the classical approach to trading day in monthly data, is inappropriate. The sum of the annual and weekly components (not displayed) contains all of the seasonality together, and corresponds in the case of monthly data to adding the trading day component to the seasonal component.

6 Conclusion

This paper proposes a methodology for studying high frequency time series with holiday patterns and multiple forms of seasonality. Structural models, with separate stochastic components for each important frequency in the data spectral density, are proposed, along with simple regressors for holidays based upon a window of activity. The proposed model for the weekly seasonal component of the daily series offers a stochastic generalization of the fixed trading day effect in monthly series. It is shown how these models correspond to a Gaussian process with fixed effects, which can be converted to stationarity through appropriate differencing. Such a model can be fitted to high frequency time series, such as the nine retail series furnished by FirstData, using the Gaussian likelihood, as implemented in the routines of *sigex*.

Once models have been fitted and assessed, the holiday effects can be captured by examining the corresponding regression coefficients, and both annual and weekly seasonality can be extracted using classical algorithms. The uncertainty of both holiday effects and seasonal adjustment can be assessed using these tools, because the variability is a by-product of the algorithms. For the retail data the trend was extremely simple, being merely a constant, and hence separation of trend and annual seasonality was trivial; more complicated cases, where a longer series requires a stochastic trend, can in principle be addressed through the same methodology, but this is left for future research.

The efficacy of the holiday modeling has some implications for seasonal adjustment internally at USCB, where analysts have speculated that newer festivals such as Black Friday and Cyber Monday do exert an impact on different facets of the retail economy. This conjecture has been authenticated by our empirical results, and such a finding was not really possible before the access facilitated by FirstData.

Findings from this work have had an immediate impact on the monthly retail series published by the USCB, thereby affirmatively answering the second question posed in the introduction – that daily data can assist with the seasonal adjustment of monthly data. In particular, the Easter Sunday regressor was introduced in the February 2017 release of X-13ARIMA-SEATS. Moreover, in

April 2017, the Easter regressor was implemented in the Monthly Retail Trade Survey production for the Building Materials and Supplies Dealers series as well as the Automobile and Other Motor Vehicle Dealers series (NAICS 4411 and NAICS 4412). The use of these new holiday regressors for monthly series have provided superior modeling in some cases, giving an indirect validation of the first question raised in the introduction, namely, that to some extent a daily retail series can function as a proxy (at least for research purposes) of a corresponding monthly retail series.

Acknowledgements We thank FirstData for provision of the series, and Palantir for delivering the data (edited for disclosure avoidance).

References

[1] Abowd, J. and Vilhuber, L. (2011) National Estimates of Gross Employment and Job Flows from the Quarterly Workforce Indicators with Demographic and Industry Detail. *Journal of Econometrics* **161**(1): 82–99.

[2] Bell, W.R. (2004) On RegComponent time series models and their applications. In *State Space and Unobserved Component Models: Theory and Applications*, eds. A.C. Harvey, S.J. Koopman, and N. Shephard, 248–283. Cambridge: Cambridge University Press.

[3] Bell, W.R. and Hillmer, S. C. (1983) Modeling time series with calendar variation. *J. Am. Stat. Assoc.* **78**, 526–534.

[4] Cleveland, W. P., & Scott, S. (2007) Seasonal Adjustment of Weekly Time Series with Application to Unemployment Insurance Claims and Steel Production. *Journal of Official Statistics* **23**(2), 209–221.

[5] De Livera, A.M., Hyndman, R.J., and Snyder, R.D. (2011) Forecasting time series with complex seasonal patterns using exponential smoothing. *Journal of the American Statistical Association* **106**(496), 1513–1527.

[6] Findley, D.F. (2005) Some recent developments and directions in seasonal adjustment. *Journal of Official Statistics* **21**, 343–365.

[7] Findley, D. F. and B. C. Monsell (2009) Modeling stock trading day effects under flow day-of-week effect constraints. *Journal of Official Statistics* **25**, 415–430.

[8] Findley, D., Wills, K., and Monsell, B. (2005) Issues in estimating Easter regressors using RegARIMA models with X-12-ARIMA. *Proceedings of the American Statistical Association.*

[9] Harvey, A.C. (1989) *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge University Press, Cambridge.

[10] Harvey, A., Koopman, S. J., & Riani, M. (1997) The modeling and seasonal adjustment of weekly observations. *Journal of Business & Economic Statistics* **15**(3), 354–368.

[11] Harvey, A.C. and Trimbur, T.M. (2003) General model-based filters for extracting trends and cycles in economic time series. *Review of Economics and Statistics* **85**, 244–255.

[12] Hyndman, R. J. and Fan, S. (2010) Density forecasting for long-term peak electricity demand. *Power Systems, IEEE Transactions on* **25**(2):1142–1153.

[13] Maravall, A. and Pérez, D. (2012) Applying and interpreting model-based seasonal adjustment – the Euro-area industrial production series. In *Economic Time Series: Modeling and Seasonality*, eds. W.R. Bell, S.H. Holan, and T.S. McElroy, 281–313. New York: CRC Press.

[14] McElroy, T. S. (2008) Matrix Formulas for Nonstationary ARIMA Signal Extraction. *Econometric Theory* **24**, 1–22.

[15] McElroy, T. S. (2017) Multivariate Seasonal Adjustment, Economic Identities, and Seasonal Taxonomy. Published online, *Journal of Business and Economics Statistics*, 1–15.

[16] McElroy, T.S. and Maravall, A. (2014) Optimal Signal Extraction with Correlated Components. *Journal of Time Series Econometrics* **6**(2), 237–273.

[17] McElroy, T. S. and Trimbur, T.M. (2015) Signal Extraction for Nonstationary Multivariate Time Series with Illustrations for Trend Inflation. *Journal of Time Series Analysis* **36**, 209–227. Also in "Finance and Economics Discussion Series," Federal Reserve Board. 2012-45 `http://www.federalreserve.gov/pubs/feds/2012/201245/201245abs.html`

[18] Pierce, D.A., Grupe, M.R., and Cleveland, W.P. (1984) Seasonal Adjustment of Weekly Monetary Aggregates: A Model-Based Approach. *Journal of Business & Economics Statistics* **2**, 260–270.

[19] Roberts, C., Holan, S., and Monsell, B. (2010) Comparison of X-12-ARIMA trading day and holiday regressors with country specific regressors. *Journal of Official Statistics*, Vol.26, No.2, pp. 371–394 .

[20] Soukup, R. and Findley, D. (2000) Modeling and model selection for moving holidays. *Proceedings of the American Statistical Association.*

[21] Tiao, G. and Tsay, R. (1983) Consistency properties of least squares estimates of autoregressive parameters in ARMA models. *Ann. Stat.* **11**, 856–871.

[22] Weinberg, J., Brown, L. D., and Stroud, J. R. (2007) Bayesian forecasting of an inhomogeneous poisson process with applications to call center data. *Journal of the American Statistical Association* **102**(480):1185–1198.

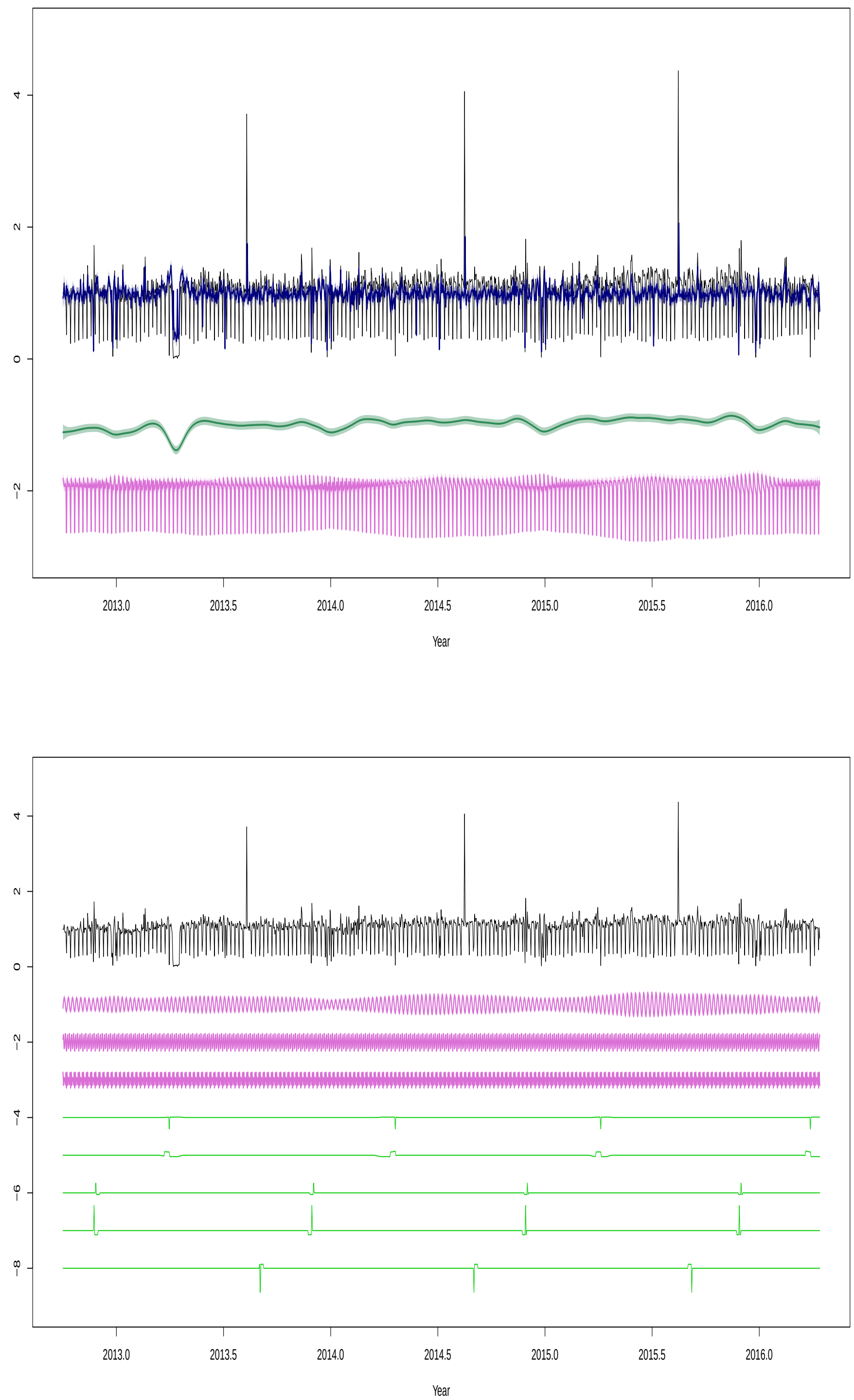
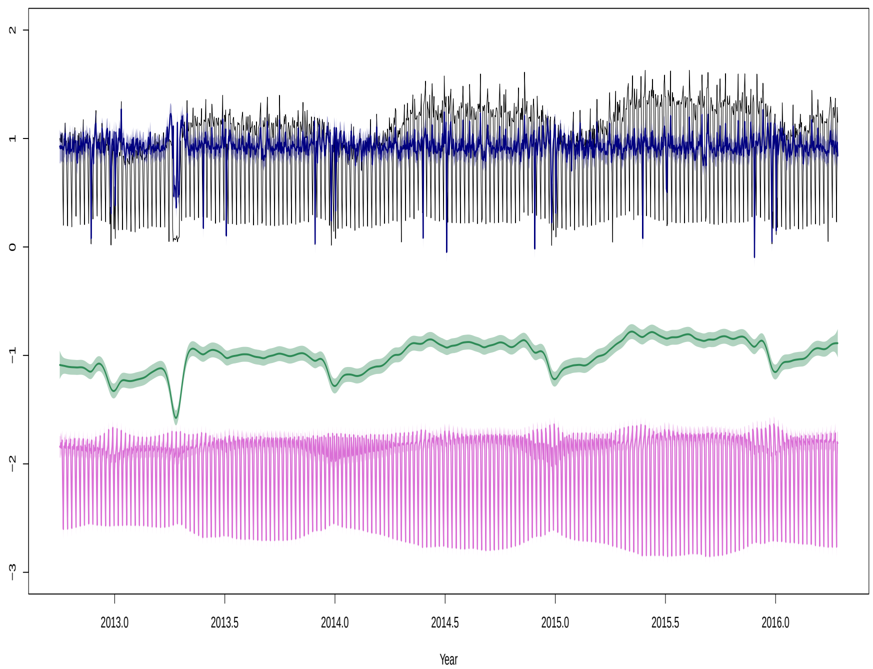


Figure 4: Signal extraction plot for Series 44311 (Appliance Stores). Upper panel: data in black, seasonal adjustment in blue, annual component in olive, weekly component in purple. Lower panel: data in black, atomic weekly components in purple, holiday effects (Easter, Cyber Monday, Black Friday and Labor Day) in green.



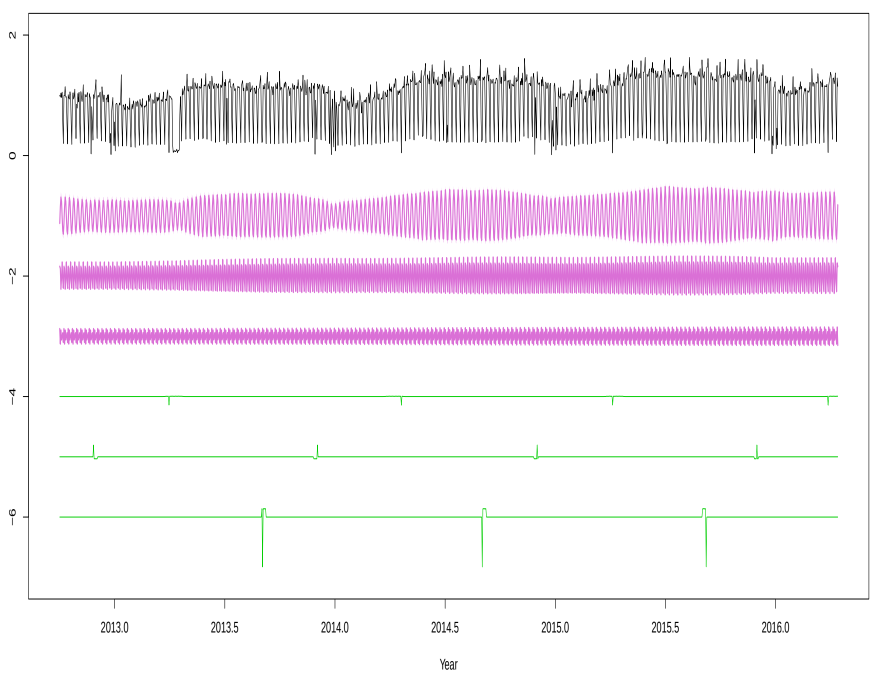
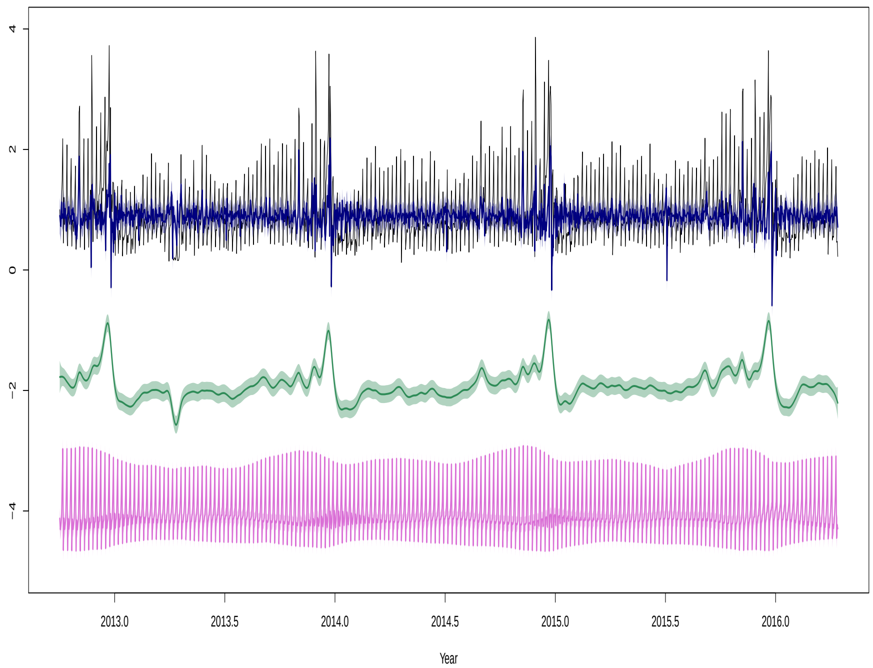


Figure 5: Signal extraction plot for Series 44411 (Home Center Stores). Upper panel: data in black, seasonal adjustment in blue, annual component in olive, weekly component in purple. Lower panel: data in black, atomic weekly components in purple, holiday effects (Easter, Cyber Monday, and Labor Day) in green.



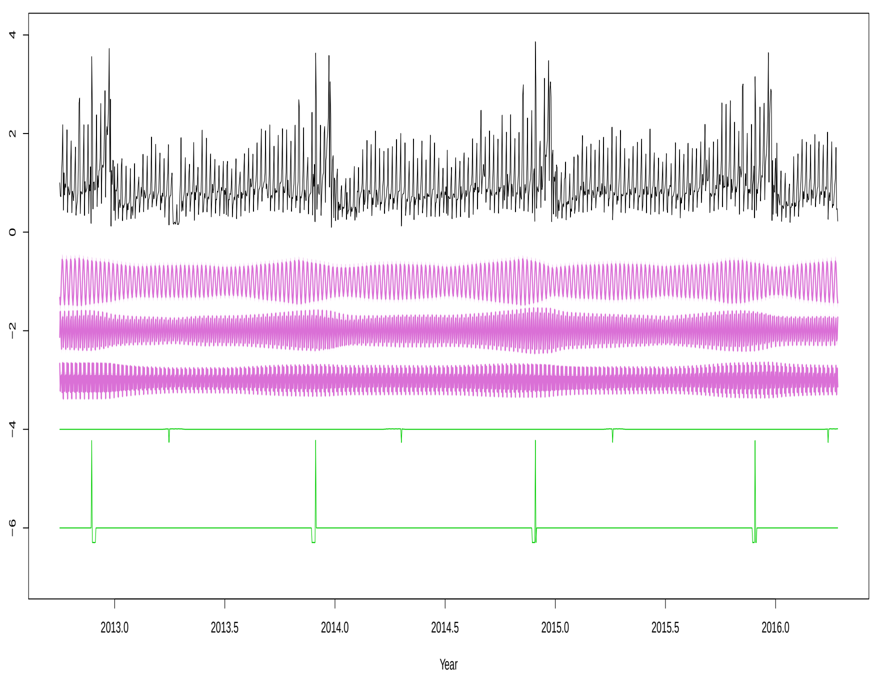


Figure 6: Signal extraction plot for Series 45211 (Department Stores). Upper panel: data in black, seasonal adjustment in blue, annual component in olive, weekly component in purple. Lower panel: data in black, atomic weekly components in purple, holiday effects (Easter and Black Friday) in green.

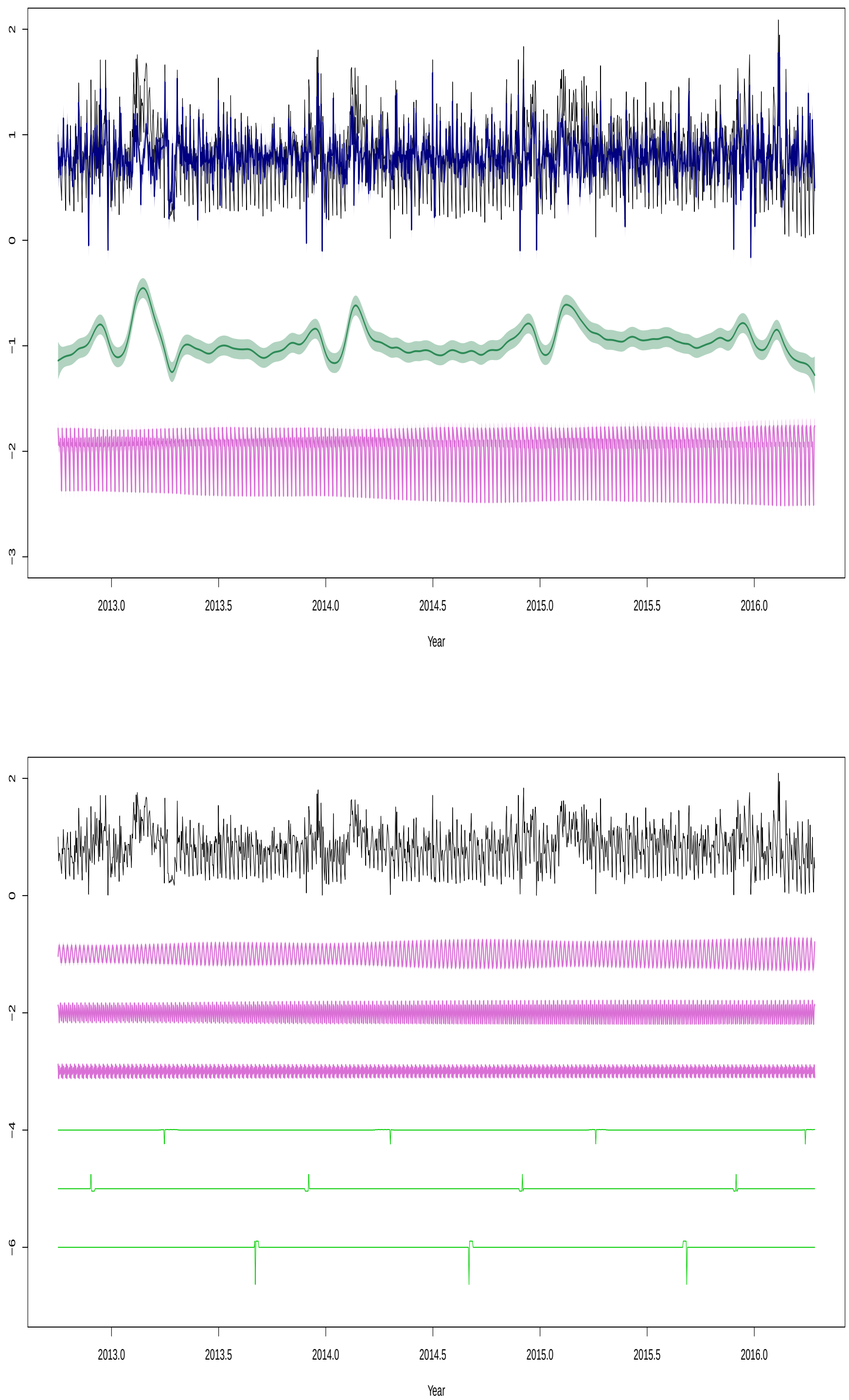


Figure 7: Signal extraction plot for Series 45291 (Warehouse Stores). Upper panel: data in black, seasonal adjustment in blue, annual component in olive, weekly component in purple. Lower panel: data in black, atomic weekly components in purple, holiday effects (Easter, Cyber Monday, and Labor Day) in green.

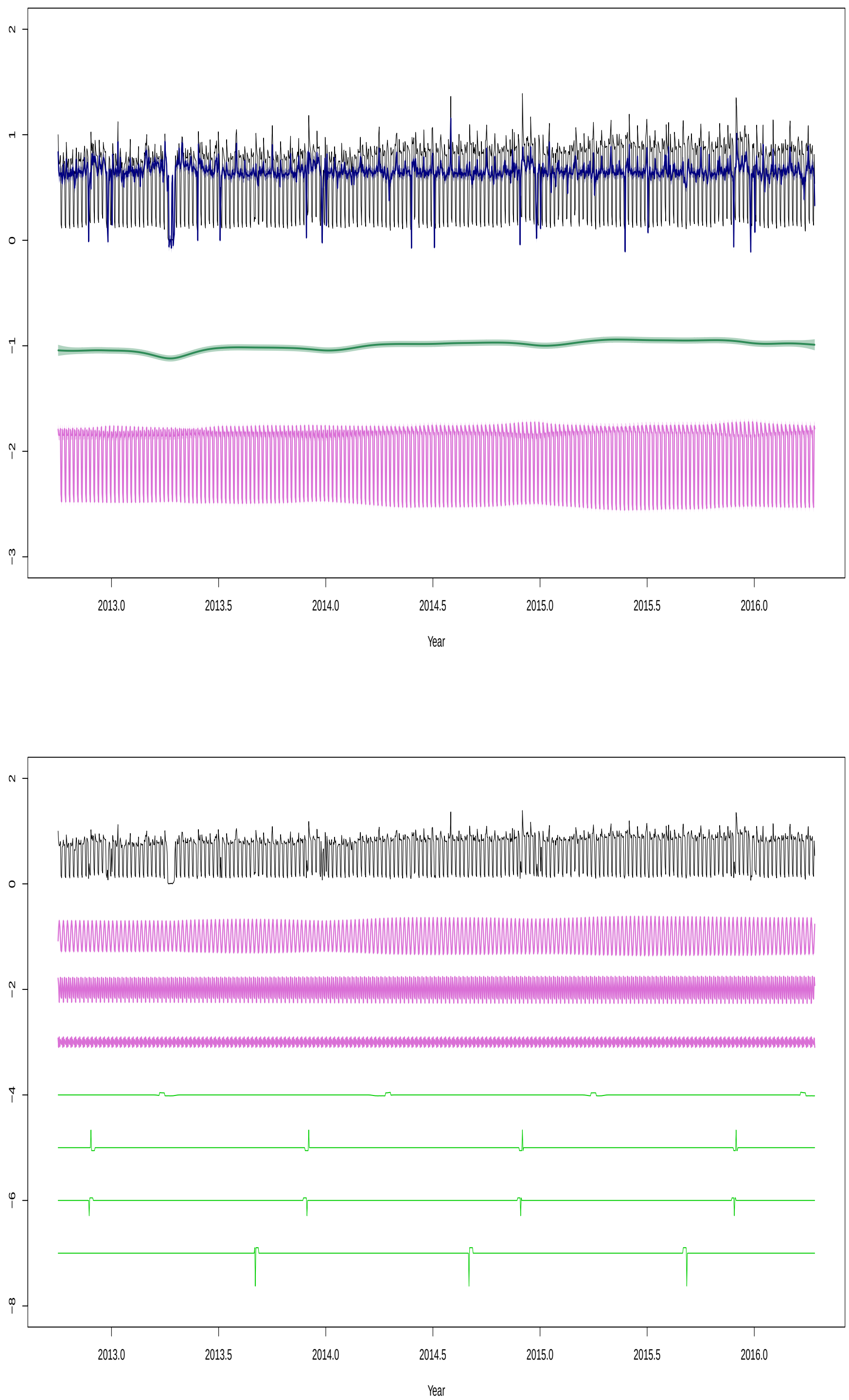


Figure 8: Signal extraction plot for Series 45411 (Electronic Shopping Stores). Upper panel: data in black, seasonal adjustment in blue, annual component in olive, weekly component in purple. Lower panel: data in black, atomic weekly components in purple, holiday effects (Easter, Cyber Monday, Black Friday, and Labor Day) in green.

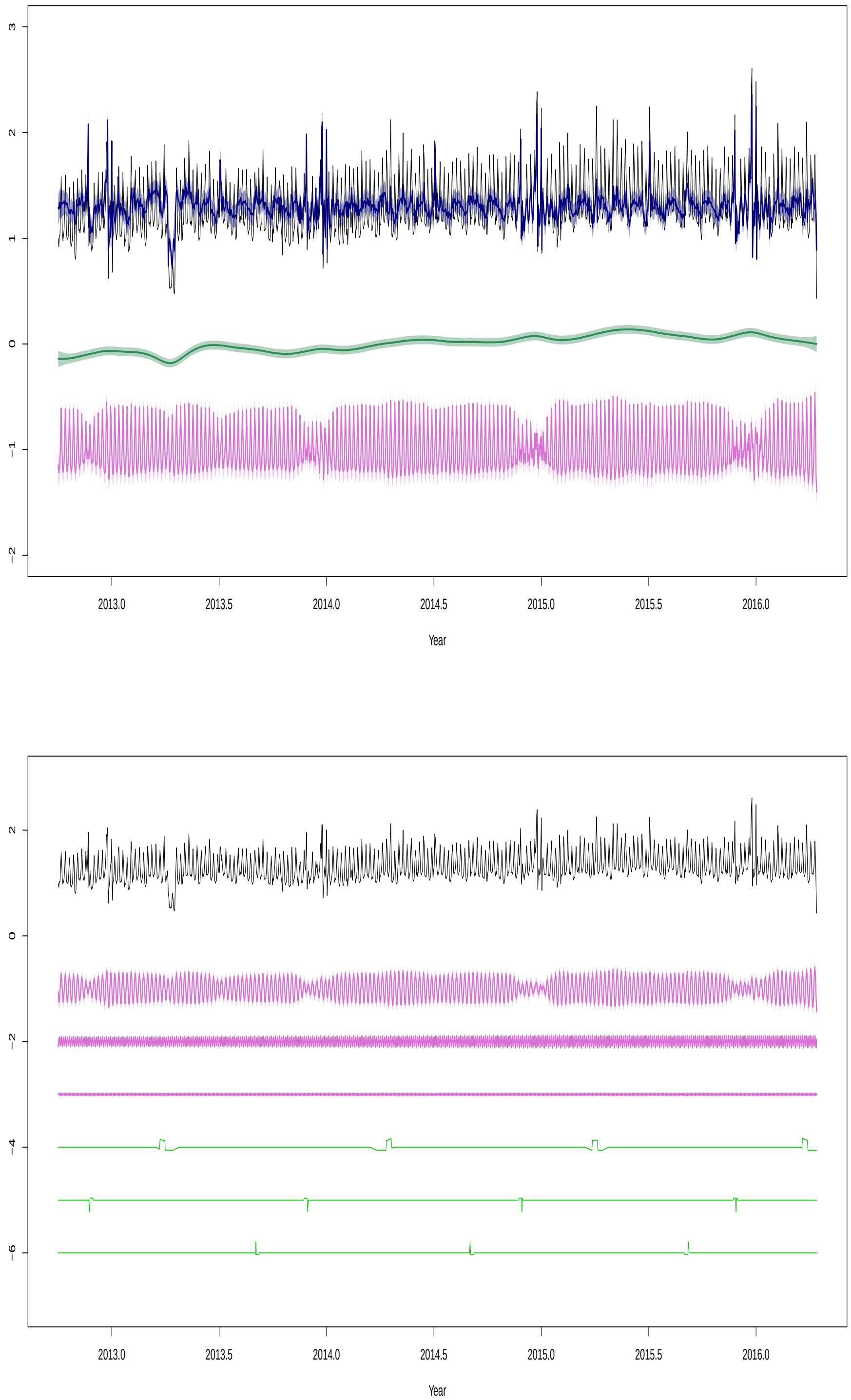


Figure 9: Signal extraction plot for Series 44511 (Supermarket Stores). Upper panel: data in black, seasonal adjustment in blue, annual component in olive, weekly component in purple. Lower panel: data in black, atomic weekly components in purple, holiday effects (Black Friday and Labor Day) in green.

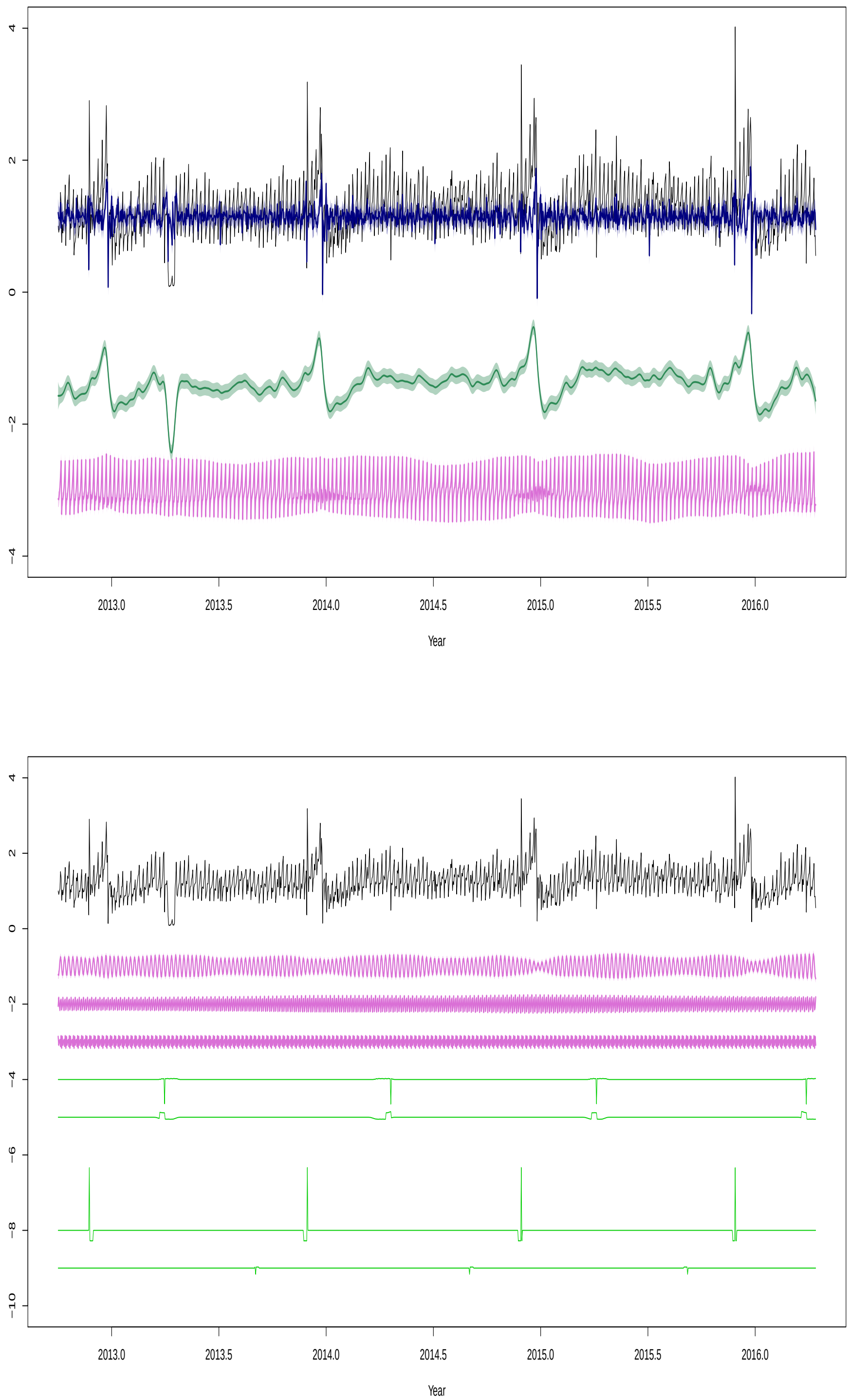


Figure 10: Signal extraction plot for Series 44814 (Clothing Stores). Upper panel: data in black, seasonal adjustment in blue, annual component in olive, weekly component in purple. Lower panel: data in black, atomic weekly components in purple, holiday effects (Easter and Black Friday) in green.

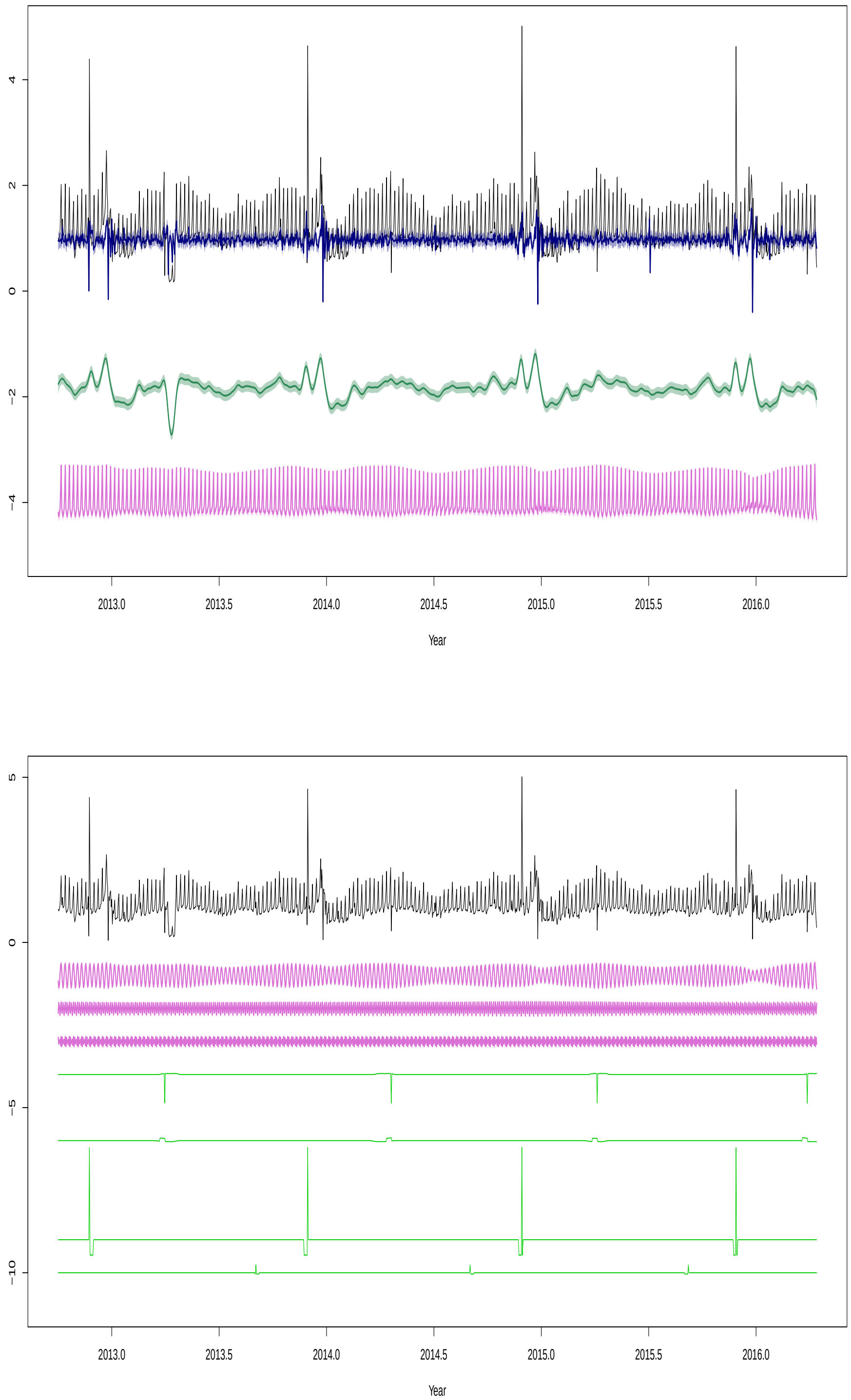


Figure 11: Signal extraction plot for Series 4482 (Shoe Stores). Upper panel: data in black, seasonal adjustment in blue, annual component in olive, weekly component in purple. Lower panel: data in black, atomic weekly components in purple, holiday effects (Easter, Black Friday, and Labor Day) in green.

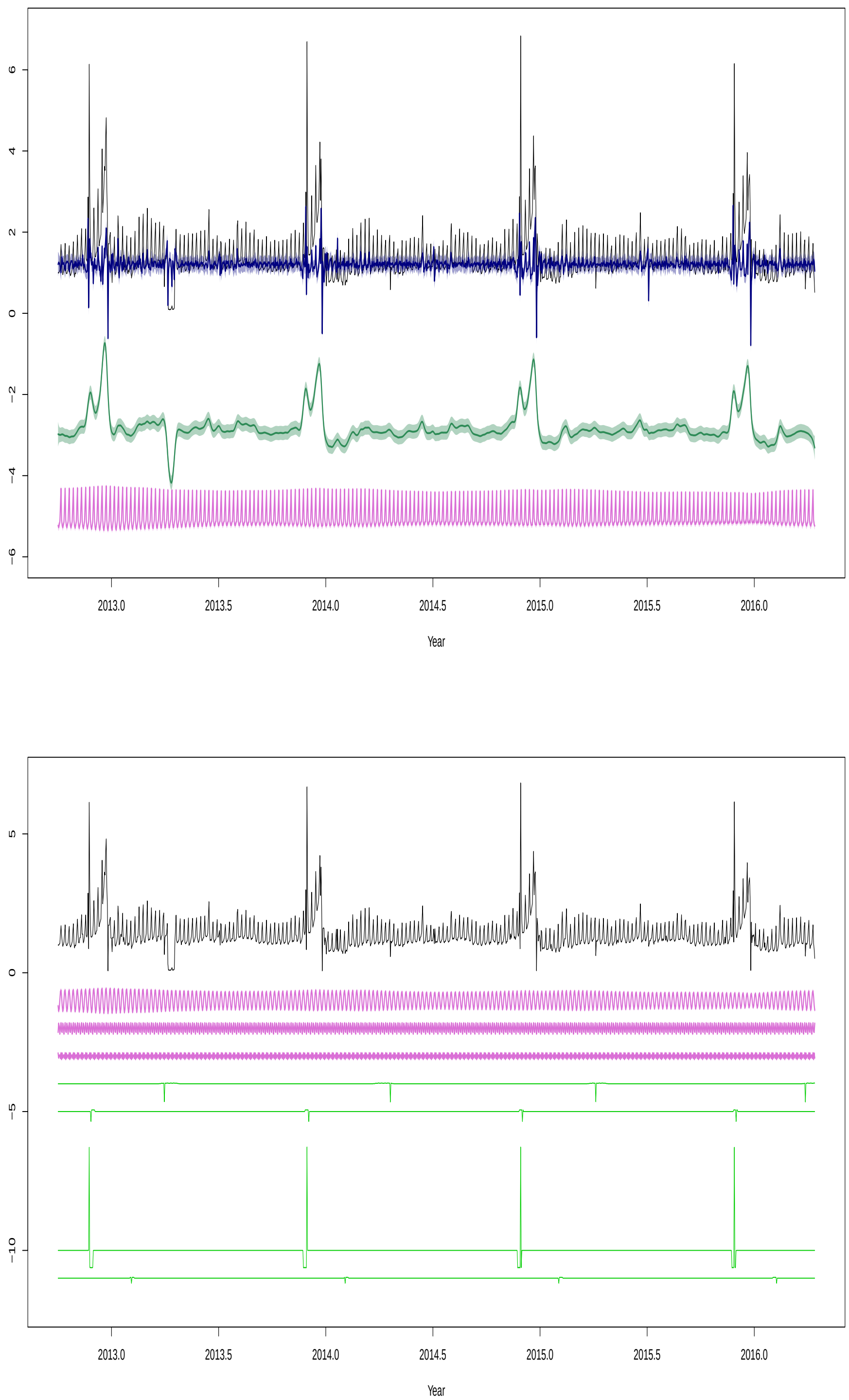


Figure 12: Signal extraction plot for Series 45111 (Sporting Goods Stores). Upper panel: data in black, seasonal adjustment in blue, annual component in olive, weekly component in purple. Lower panel: data in black, atomic weekly components in purple, holiday effects (Easter, Cyber Monday, and Black Friday) in green.