Multidimensional Inequality: Measurement and Analysis using the American Community Survey

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Abstract

This paper creates a multidimensional inequality measure using the American Community Survey.[1] Rather than limiting the study of inequality to income, this measure also includes inequality in leisure, health, education, housing, and vehicle ownership. There are two ways that inequality in different dimensions have been studied. The first is the individual approach. This involves analyzing and discussing each dimension's inequality one at a time. The second is an aggregation approach in which a single aggregated measure of inequality is produced. The analysis in this paper focuses on three main areas. First, to examine how multidimensional inequality results change when choices of parameters change. The varying parameters in this case are the degree of substitutability among the dimensions and the degree of aversion to inequality. Second, to compare inequality using the individual approach and the aggregate approach to each other as well as to traditional unidimensional income inequality measures. Third, to examine how multidimensional inequality varies over time and by state.

[1] The Census Bureau reviewed this data product for unauthorized disclosure of confidential information and has approved the disclosure avoidance practices applied to this release. DRB-FY19-ROSS-B0062.

Introduction

Income is not the only thing people care about in relative terms. Keeping up with the Joneses can involve leisure time, health, education, housing, and vehicle ownership as well as income. There are two ways that inequality in different dimensions have been studied. The first is the individual approach. This involves analyzing and discussing each dimension's inequality one at a time. The advantage of this approach is that there is a wealth of information. However, it is not easy to compare one area to another using this approach unless the special case arises in which all inequality measures are higher in one area than in another.

The second is an aggregation approach. The aggregation methods used in the literature involve a two-step process. In the first step, the different dimensions are aggregated together using a standardization method so that each dimension is measured in the same units. In the second step, inequality measures are calculated using this aggregated value.

While some results are presented using the individual approach, the majority of the analysis in this paper uses the aggregation approach. When using the aggregation approach, there are a number of different choices that must be made in order to calculate multidimensional inequality. These choices are what dimensions to include, the dataset to use, the standardization method and inequality measure, the degree of substitutability among the dimensions, how each dimension is weighted, and the degree of aversion to inequality.

The multidimensional inequality measure constructed in this paper includes income along with the five other dimensions listed in Table 1. Most of the data used to measure these dimensions were from the American Community Survey 1-year estimates for the years 2008[2] through 2017. The health dimension was based on predicted health status using data from the Current Population Survey Annual Social and Economic Supplement.

Table 1: Dimensions in Multidimensional Inequality Measure

Inequality type	How it's measured
Income	Household income adjusted for the size and composition of the household and for differences in state price levels as measured by regional price parities
Education	Average number of years of education per each person in the household age 25 and over
Health	Average predicted health status for the members of the household
Leisure	Average time in a typical week that is not spent sleeping, working, or commuting by each person in the household between the ages of 25 and 64
Vehicle ownership	Number of vehicles owned per adult in the household
Housing	Number of rooms per person in a house or apartment

[2] The earliest ACS available is 2005. However, data limitations, namely the health dimension, limits analysis to 2008.

The Maasoumi method[3] was used to create the multidimensional inequality measure. This is a two-step method that has often been used in the literature. In the first step, each dimension was standardized so that they can be aggregated together using similar units. The standardization method in this paper involves dividing each value in a dimension by the mean value of that dimension. The dimensions were then added together using equal weights for each dimension. The second step was to use the Atkinson index or the Multidimensional Gini on this aggregated value. The Atkinson Index requires one to choose the values of two parameters: the degree of substitutability among the dimensions and the degree of aversion to inequality in society. Several different values of these parameters were examined.

The analysis in this paper focuses on three main areas: first, the examination of the sensitivity of multidimensional inequality results due to choices of parameters for the degree of substitutability among the dimensions and the degree of aversion to inequality; second, comparisons of the individual and the aggregate approaches to each other as well as to traditional income inequality measures; and third, an examine of variations in multidimensional inequality by state and over time.

Literature Review

Just as the literature has suggested that poverty may more completely be described using multiple dimensions, research on inequality in areas other than income has been on the rise recently. A number of studies on multidimensional inequality are compared in Table 2, the vast majority of which were published in the last ten years. These studies differ in the datasets, the dimensions, the standardization and weighting methods, the inequality measures, and the inequality aversion and dimensional substitutability parameters.

For studies of multidimensional inequality in the United States, most authors have used the Panel Study of Income Dynamics (PSID). No other datasets were used by more than one group of authors. Based on this literature review, no one has used the American Community Survey to study multidimensional inequality.

While the measurement of dimensions may have varied considerably in this literature review, the choice of dimensions were fairly consistent. Education was included in every study, while income and health were included in all but one. Furthermore, all but one of the studies included three dimensions. The only reasons given for the choice of number of dimensions were data limitations and the interests of the authors.

The standardization method chosen was divided between mean standardization and standardization by subtracting off the minimum value and dividing by the range of the data. Both of these methods return a value without any units which allows for different dimensions to be aggregated together. The first method results in values with a mean of 1 for each dimension and the second method results in values between 0 and 1 for each dimension.

Equal weighting of dimensions was used in every paper reviewed in this literature review. Several papers (Maasoumi and Nickelsburg 1988; Rodde and Guest 2013; Rohde and Guest 2017) used other weighting schemes, along with equal weights, such as data driven weights or weights based on

[3] Maasoumi, E., "The Measurement and Decomposition of Multidimensional Inequality", Econometrica, 54, 991-7, 1986.

author's intuition. The consensus overall seemed to be to weight dimensions equally but allow the substitutability among the dimensions to vary.

Two basic types of inequality measures were used in the literature: Generalized entropy, in which Atkinson is a specialized form, and Gini. Generalized entropy was the preferred specification in the literature due to its flexible forms, but the Gini index also offers ease of interpretation and general familiarity.

The final area in which the literature differed was in the selection of parameters. The magnitude of the dimensional substitutability parameters differed, but most studies included at least one positive value, zero, and at least one negative value. The aversion parameters differed as well, however part of that difference was based on whether the parameter was an α or an ε, where ε=1- α. In general, studies included low and moderate values of inequality aversion.

Table 2: Comparisons of the Multidimensional Inequality Literature

Paper	Dimensions	Substitution parameters	Dataset	Aversion parameters[1]	Standardization method	Weighting scheme	Measure
Maasoumi and Nickelsburg (1988)	Income (I), net housing equity, and education (E)	-2,-1,-1/2, 0	PSID	0, -1	Divide by mean	Multiple weights	Generalized entropy
Lugo (2007)	I, health (H), and E	-20, -4, 0, .333, .5, 1	Argentina Permanent Household Survey	-2, -1, 0, .333, .5, 1	Subtract minimum and divide by range	Equal	Generalized entropy
Seth (2009)	I, H, and E	-1	2000 Mexican Census	-3	Subtract minimum and divide by range	Equal	Generalized entropy
Aristei and Bracalente (2011)	I, H, and E	-5, -2, -1, 0, .5, 1	IT-SILC	.3, 1, 1.5, 2, 2.5, 3	Divide by mean	Equal	Atkinson
Decancq and Lugo (2012)	I, H, and E	0	Russian Longitudinal Monitoring Survey	2, 5	Divide by mean	Equal	Gini index
Jorda (2013)	I, H, and E	10,1,0, -10	International Human Development Indicators	1,0,-5	Subtract minimum and divide by range	Equal	Generalized entropy
Sial et al (2015)	H, E, and expenditure	0	Household Integrated Economic Survey	2, 5	Divide by mean	Equal	Gini index
Rohde and Guest (2013)	I, H, and E	0, -1	PSID	1	Divide by mean	Multiple weights	Generalized entropy
Rohde and Guest (2017)	I, H, E, and leisure (L)	1, .5, and 0	PSID, SOEP, and HILDA	0, 1	Divide by mean	Multiple weights	Generalized entropy
This paper	I, H, E, L, vehicle ownership, and housing	1,0,-1, and -2	ACS	½,1,2,3	Divide by mean	Equal	Atkinson and Gini index

Note: [1] For Atkinson and Multidimensional Gini, the aversion parameter is ε. For Generalized Entropy, the aversion parameter is α. These two parameters are related to each other by ε=1-α.

Methodology

A. Dimensions

As discussed in the literature review, previous research has mainly included income, health, and education as the dimensions of a multidimensional inequality measure. Expenditures, leisure, and housing were also included in the literature in a few instances. Some form of each of these dimensions were included in this paper's definition of multidimensional inequality. The six dimensions used are described in detail below.

1) Income

The income variable used was total pre-tax household income with two adjustments. Income was adjusted for the size and composition of the household using a three-parameter equivalence scale used by the Census Bureau in the calculation of the supplemental poverty measure:[4]

One and two adults: $scale = adults\text{^}0.5$
Single parents: $scale = (adults\ +\ 0.8\ \times\ first\ child\ +\ 0.5\ \times\ other\ children)\text{^}0.7$
All other families: $scale = (adults\ +\ 0.5\ \times\ children)\text{^}0.7$

where *adults* is the number of adults in the household, *first child* is equal to one if the household has at least one child, *other children* is equal to the number of children in the household minus one, and *children* is the number of children in the household. Income was divided by this scale variable to get a measure of equivalence adjusted household income. This is done because resources are shared among people in a household resulting in economies of scale and children use less resources than adults. Income was also adjusted for state cost of living differences using state-level regional price parities created by the Bureau of Economic Analysis.[5]

2) Education

To create the education variable, each person in the household aged 25 and older is assigned the number of years of schooling that they received. The number of years of schooling were then averaged across all members of the household aged 25 and over in order to create an average number of years of schooling for each household.[6]

3) Health

Ideally, the actual health status of each person in the household would be used to measure health inequality. Since this information is not available, reported health status, infant mortality rates, and life expectancy rates have been used as proxies. Reported health status is not available in the ACS. However, reported disabilities[7] are available in the ACS and both reported health status[8] and disabilities are available in the CPS ASEC. This allows for the assignment of predicted health status in the ACS using information from the CPS ASEC: health status was regressed on age, gender, and six disability dummy variables in order to construct predicted health status. Since predicted health status is worse as the value increases, the inverse of the predicted health status is used in order to be consistent with the other dimensions.

[4] Fox, Liana. 2017. "The Supplemental Poverty Measure: 2016". Current Population Reports. U.S. Census Bureau.
[5] For more information on regional price parities, see https://www.bea.gov/newsreleases/regional/rpp/rpp_newsrelease.htm.
[6] Households with all members under 25 years of age did not have a value for education: 3.69% of households.
[7] There are six disabilities a person can report in the ACS and CPS ASEC: Hearing difficulty, vision difficulty, difficulty going out, difficulty dressing, physical difficulty, and difficulty remembering.
[8] Health status in the CPS ASEC is reported as: 1-excellent, 2-very good, 3-good, 4-fair, and 5-poor.

4) Leisure

Leisure time, for the purposes of this paper, is time that people could theoretically use as they please. It is time not spent working, commuting, or sleeping in a week. Each person was assigned 8 hours of sleep per day. These values were only calculated for prime age workers (people aged 25 to 64). To create a household measure, these leisure values were averaged for the members of the household aged 25 to 64.[9]

5) Vehicle ownership

Income in and of itself is important only as a means to an end. People may have different incomes, but do they have different abilities to spend? Ideally, a measure of durable and non-durable goods purchased would show consumption inequality. That data is not available in the ACS. Instead, the number of vehicles owned per adult in the household is used.

6) Housing

The quality of a residence is important for the enjoyment of life and for the safety and security of the family. There is no measure of housing quality in the ACS but housing can be measured as the number of rooms per person in a house or apartment. While this is not a measure of the quality of housing, it is an indicator of the extent of overcrowding.

B. Maasoumi Method

The Maasoumi method is the usual method used in the literature to generate a multidimensional inequality index.[10] In the first step, dimensions need to be standardized so that they can be combined with one another into a single multidimensional value. In the second step, an inequality measure is used on this value.

1) Standardization and aggregation

In the literature, dimensions have been standardized in two ways. One way is to divide each dimension by its mean. The advantages in this case are that each dimension has a mean equal to one and values are unitless. A disadvantage is that this method doesn't make sense to use with negative values. A second way is to subtract the minimum from each value and then divide by the range of values. Means are different in this case but all dimensions range from zero to one. In this paper, the first method of standardization is used.

The equation used to aggregate the six dimensions into a single well-being measure is listed below:

$$S_i = (\Sigma_k \, w_k Z_{ik}^{\beta})^{1/\beta} \quad , \beta \neq 0$$

[9] Households with all members under 25 years of age and/or over 64 years of age did not have a value for leisure: 40.89% of households.

[10] Maasoumi (1986).

$$S_i = \Pi_k (Z_{ik})^{w_k} \quad , \beta=0$$

where S_i is the well-being measure of person *i*, w_k is the weight of dimension *k*, $Z_{ik} = f(X_{ik})$, X_{ik} is the value of dimension *k* for person *i*, f() is the standardization function, and β is the substitutability parameter among the dimensions.[11]

2) Inequality measure

In the literature, three main inequality measures were used in the second step: Multidimensional Gini, Generalized entropy, and Atkinson. While multidimensional Gini results are reported and discussed, the Atkinson measure is the main measure used in this paper for three reasons. First, it is a measure that easily and clearly allows for the use of different inequality aversion parameters. Second, generalized entropy and Atkinson measures are related to one another. A generalized entropy measure can be transformed into an Atkinson measure using the formula A = 1 – e^(-GE(α)) with ε=1-α. Since the inequality aversion parameter in Atkinson is more intuitive than in the generalized entropy measures, the Atkinson Index is used in this paper. While generalized entropy results are not presented, some multidimensional Gini results are presented in the results section. Third, Atkinson's measures have not been used often in the multidimensional inequality literature. The multidimensional Atkinson inequality measure is listed below:

$$I^A = 1 - \left[\frac{1}{n} \Sigma_i \left(\frac{S_i}{\mu}\right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \quad , \varepsilon \neq 1$$

$$I^A = \frac{1}{n} \Sigma_i \log \left(\frac{\mu}{S_i}\right) \quad , \varepsilon=1$$

where μ is the mean of the well-being measure, S_i, and ε is the inequality aversion parameter.

C. Substitutability

When dealing with multiple dimensions, it is important to decide how the dimensions interact with each other. If low levels of one dimension can be perfectly compensated by high levels of another, then the dimensions are perfect substitutes (β=1). In the other extreme, if dimensions cannot compensate for each other than they are perfect complements (β→-∞). This paper examines several intermediate degrees of substitutability along with perfect substitutes: 1, 0, -1, and -2.

D. Inequality Aversion

Inequality aversion is a measure of how society feels about income or well-being distributions. Putting that feeling into equation form, the larger the aversion to inequality, the more sensitive the inequality measure is to changes at the lower part of the well-being distribution. Inequality aversion with the Atkinson Index is straightforward. If ε=0, there is no aversion to inequality and the Atkinson Index is zero. As ε increases, inequality aversion increases and the Atkinson Index increases. In this

[11] There are two dimensions, Education and Leisure, for which not all households have a value. For households missing a leisure component for example, their multidimensional inequality value is based only on five dimensions. Each dimension would be weighted 20 percent instead of 16.7 percent.

paper, the inequality aversion parameter takes on the values .5, 1, 2, and 3. This is done in order to show what happens to inequality at relatively low and relatively high levels of inequality aversion.

E. Weighting

In the literature, nearly all researchers have used equal weighting of dimensions. This is generally done in order to not make any value judgements about what dimension may be more important than another dimension. In a few cases, data driven weights have been used along with equal weights for robustness purposes. The data driven weights are based on the idea that highly correlated dimensions may be double counting people. Therefore, set dimension *k*'s weight as proportional to the inverse of the sum of the absolute values of the correlations of dimension *k* with each of the other dimensions. The weights for the two different weighting schemes are listed in table 3. The biggest differences for data driven weights are that leisure gets a substantially larger weight while housing gets a substantially smaller weight than with equal weights.

Table 3: Weighting Schemes - 2017

Weighting scheme	Income	Education	Health	Leisure	Vehicle ownership	Housing
Equal	0.167	0.167	0.167	0.167	0.167	0.167
Data driven	0.142	0.147	0.172	0.243	0.174	0.121

Source: Author's calculations, 2017 American Community Survey.

F. Data

The data used in this paper comes from the American Community Survey (ACS) 1-year estimates for 2008, the earliest available sample that includes all the data needed for this paper, to 2017, the latest available. The ACS is the best source of sub-national economic, social, and employment characteristics, it has a large sample size which allows for breakdowns by demographic variables and by small geographies, and it has not been used, based on this literature review, to measure multidimensional inequality. Table 4 presents descriptive statistics for the six dimensions for 2017.

Table 4: Descriptive statistics - 2017

Dimension	Mean	Standard Deviation	Minimum	Maximum	Number of Households
Income	$49,910	$59,890	$0	$3,189,000	120,100,000
Education	13.76 years	2.81 years	0 years	20 years	115,700,000
Health[12]	0.44	0.08	0.20	0.65	120,100,000
Leisure	67.19 hours	10.51 hours	0 hours	111 hours	70,890,000
Vehicle ownership	0.98 cars	0.55 cars	0 cars	6 cars	120,100,000
Housing	3.03 rooms	1.87 rooms	0.05 rooms	81 rooms	120,100,000

Source: U.S. Census Bureau, 2017 American Community Survey.

[12] The Health variable does not have a straight forward interpretation since it is the inverse of predicted values.

Since each of the dimensions appears to be related to income, it is reasonable to wonder whether a multidimensional inequality measure is actually needed as opposed to a simple income inequality measure. Simple correlations of the dimensions are presented in Table 5. While there is a somewhat strong positive linear correlation between income and education, the other dimensions are significantly less strongly correlated with income. The dimensions are also not strongly positively correlated with each other. Therefore, there is some benefit to a multidimensional measure to describe inequality.

Table 5: Simple Correlations among Six Dimensions – 2017

Dimension	Income	Education	Health	Leisure	Vehicle ownership	Housing
Income	1	0.3191	0.0138	-0.2083	0.1399	0.1409
Education		1	0.1353	-0.0915	0.1271	0.1207
Health			1	-0.0453	0.0833	-0.4118
Leisure				1	-0.0786	-0.0550
Vehicle ownership					1	0.2369
Housing						1

Source: Author's calculations, 2017 American Community Survey.

Results

In the literature, multidimensional inequality is discussed in two ways: the individual method and the aggregate method. The individual method involves examining and discussing the different dimensions individually. The inequality measures used for each dimension are the Gini Index and the Atkinson Index with three different levels of inequality aversion; .5, 1, and 2. The higher the number, the more aversion to inequality in society. In practice this means that incomes at the lower end of the distribution are given more weight.

Individual Approach

In Table 6, inequality results for the six dimensions are presented for each measure for the year 2017, the year 2008, and the percentage change in each measure from 2008 to 2017. There are three main takeaways from this table. First, in both years, income is the most unequal dimension and leisure is the most equal dimension using the Gini Index, the ATK0.5 and the ATK1. While leisure is still the most equal dimension when using the ATK2, vehicle ownership is the most unequal dimension. Second, in both years and for all dimensions, inequality in a dimension increases as aversion to inequality increases.

Third, it is possible to say that income inequality increased from 2008 to 2017, but what about overall inequality? According to the Gini Index, income, vehicle ownership, and housing inequality increased over this time period while education, health, and leisure inequality decreased. Without making a value judgement about what dimension is more important than another, nothing can be said about the change in overall inequality using the individual method. Furthermore, while the results for the Atkinson measures are similar, there are several dimensions that did not change significantly from 2008 to 2017 and one dimension (Leisure) that increased in inequality for the ATK2 which decreased in inequality for the Gini Index.

Table 6: Inequality by individual Dimensions - 2008 and 2017

		Gini		ATK0.5		ATK1		ATK2	
	Dimension	**Est.**	**Std. Err.**	**Est.**	**Std. Err.**	**Est.**	**Std. Err.**	**Est.**	**Std. Err.**
2017	Income	0.4522	0.0004	0.1727	0.0011	0.3792	0.0009	0.5414	0.0007
	Education	0.1053	0.0001	0.0134	0.0003	0.0333	0.0003	0.1365	0.0010
	Health	0.1035	0.0001	0.0089	0.0001	0.0183	0.0001	0.0381	0.0002
	Leisure	0.0811	0.0001	0.0067	0.0002	0.0142	0.0002	0.0339	0.0004
	vehicle ownership	0.2612	0.0003	0.0904	0.0008	0.2536	0.0011	0.6284	0.0008
	Housing	0.3212	0.0002	0.0825	0.0011	0.1752	0.0010	0.3032	0.0010
2008	Income	0.4418	0.0004	0.1653	0.0011	0.3611	0.0009	0.5257	0.0007
	Education	0.1082	0.0001	0.0142	0.0003	0.0346	0.0003	0.1351	0.0009
	Health	0.1050	0.0001	0.0096	0.0001	0.0200	0.0001	0.0426	0.0002
	Leisure	0.0814	0.0001	0.0067	0.0002	0.0141	0.0002	0.0329	0.0004
	vehicle ownership	0.2562	0.0003	0.0858	0.0007	0.2313	0.0009	0.5627	0.0007
	Housing	0.3176	0.0002	0.0819	0.0008	0.1710	0.0007	0.2951	0.0008
Percent change	Income	*2.36	0.13	*4.49	0.97	*5.00	0.36	*2.99	0.19
	Education	*-2.72	0.14	*-5.08	2.80	*-3.90	1.13	1.02	0.96
	Health	*-1.41	0.08	*-7.61	2.01	*-8.70	0.87	*-10.73	0.57
	Leisure	*-0.30	0.17	0.35	5.04	0.79	2.20	*3.12	1.67
	Vehicle ownership	*1.96	0.18	*5.36	1.21	*9.64	0.62	*11.67	0.21
	Housing	*1.44	0.10	0.72	1.67	*2.45	0.65	*2.72	0.42

*** percent change in inequality measure is statistically significant at the 90 percent confidence level.**
Source: Author's calculations, 2008 and 2017 American Community Surveys.

In table 7, inequality results between New York and California are compared in order to further demonstrate the difficulty in ranking overall inequality between geographic areas using the individual method. While income inequality, health inequality, and vehicle ownership inequality are higher in New York than in California, the opposite is true for education inequality and housing inequality. Furthermore, there is no significant difference in leisure inequality between the two states.

Table 7: Comparison of Inequality between New York and California: 2017

	New York ATK2		California ATK2		Difference	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
Income	0.5900	0.0024	0.5729	0.0019	*-0.0171	0.0030
Education	0.1679	0.0045	0.2021	0.0021	*0.0342	0.0052
Health	0.0369	0.0007	0.0350	0.0005	*-0.0020	0.0009
Leisure	0.0345	0.0016	0.0335	0.0011	-0.0009	0.0020
Vehicle ownership	0.7828	0.0010	0.5838	0.0020	*-0.1990	0.0022
Housing	0.3190	0.0023	0.3404	0.0017	*0.0215	0.0028

*** difference is statistically significant at the 90 percent confidence level**
Source: Author's calculations, 2017 American Community Survey.

Aggregate Approach

While the individual method is useful in understanding inequality in individual dimensions, it is less useful in comparing overall inequality changes over time or between geographic areas. In order to do this, the aggregate method is used. The aggregate method involves aggregating the individual

dimensions into a single inequality measure. In this method, Atkinson measures and the Gini index are used once again.

In Table 8, Atkinson measures are presented with two varying parameters. The first is the inequality aversion parameter, ε, which ranges from little aversion, .5, to significant aversion, 3. The second is the dimensional substitution parameter, β, which ranges from perfect substitutes, 1, to limited substitutability, -2.

Table 8: Multidimensional Inequality using the Atkinson Index - 2008 and 2017

		Inequality aversion --------> more aversion							
		ε = .5		ε = 1		ε = 2		ε = 3	
2017		Est	Std. Err	Est	Std. Err	Est	Std. Err	Est	Std. Err
Substitutability	β = 1	0.0193	0.0003	0.0380	0.0004	0.0717	0.0004	0.1057	0.0001
↓	β = 0	0.0241	0.0003	0.0516	0.0005	0.1104	0.0005	0.1821	0.0001
Less	β = -1	0.0462	0.0004	0.1088	0.0007	0.2509	0.0007	0.4130	0.0002
substitutable	β = -2	0.0654	0.0004	0.1635	0.0008	0.3765	0.0007	0.5744	0.0002
2008									
Substitutability	β = 1	0.0193	0.0002	0.0380	0.0003	0.0715	0.0003	0.1057	0.0002
↓	β = 0	0.0233	0.0003	0.0497	0.0004	0.1060	0.0004	0.1739	0.0001
Less	β = -1	0.0418	0.0003	0.0961	0.0006	0.2172	0.0006	0.3581	0.0002
substitutable	β = -2	0.0591	0.0004	0.1435	0.0007	0.3240	0.0006	0.5033	0.0002
Percent change									
Substitutability	β = 1	0.19	2.00	0.10	1.46	0.20	0.75	Z	0.95
↓	β = 0	*3.61	1.80	*3.88	1.31	*4.22	0.65	*4.70	0.77
Less	β = -1	*10.71	1.28	*13.14	0.94	*15.52	0.43	*15.32	0.32
substitutable	β = -2	*10.64	1.03	*14.00	0.77	*16.18	0.31	*14.13	0.14

*** percent change in inequality measure is statistically significant at the 90 percent confidence level**
Z represents or rounds to zero
Source: Author's calculations, 2008 and 2017 American Community Surveys.

Looking at the individual years, the inequality results vary greatly with choice of parameters. Multidimensional inequality increases as aversion to inequality increases and as dimensions become less substitutable. If there is little aversion to inequality and dimensions are perfect substitutes, it appears that there is very little inequality in society in 2017, 0.0193. However, if dimensions are less substitutable for one another and there is a fair amount of aversion to inequality, it appears that there is significant inequality in society in 2017, 0.5744.

Looking between years, two things were readily apparent. First, there was no significant change in any of the inequality measures that had perfectly substitutable dimensions. Second, all inequality measures with at least some complementarity of dimensions showed an increase over time.

In the literature, the second type of multidimensional inequality measures used are multidimensional Gini Indexes. In table 9, Gini indexes with moderate aversion to inequality and different levels of dimensional substitutability are presented for 2008 and 2017. The Gini index increases as dimensional substitutability decreases for both years. Inequality increases over the time period for each inequality measure.

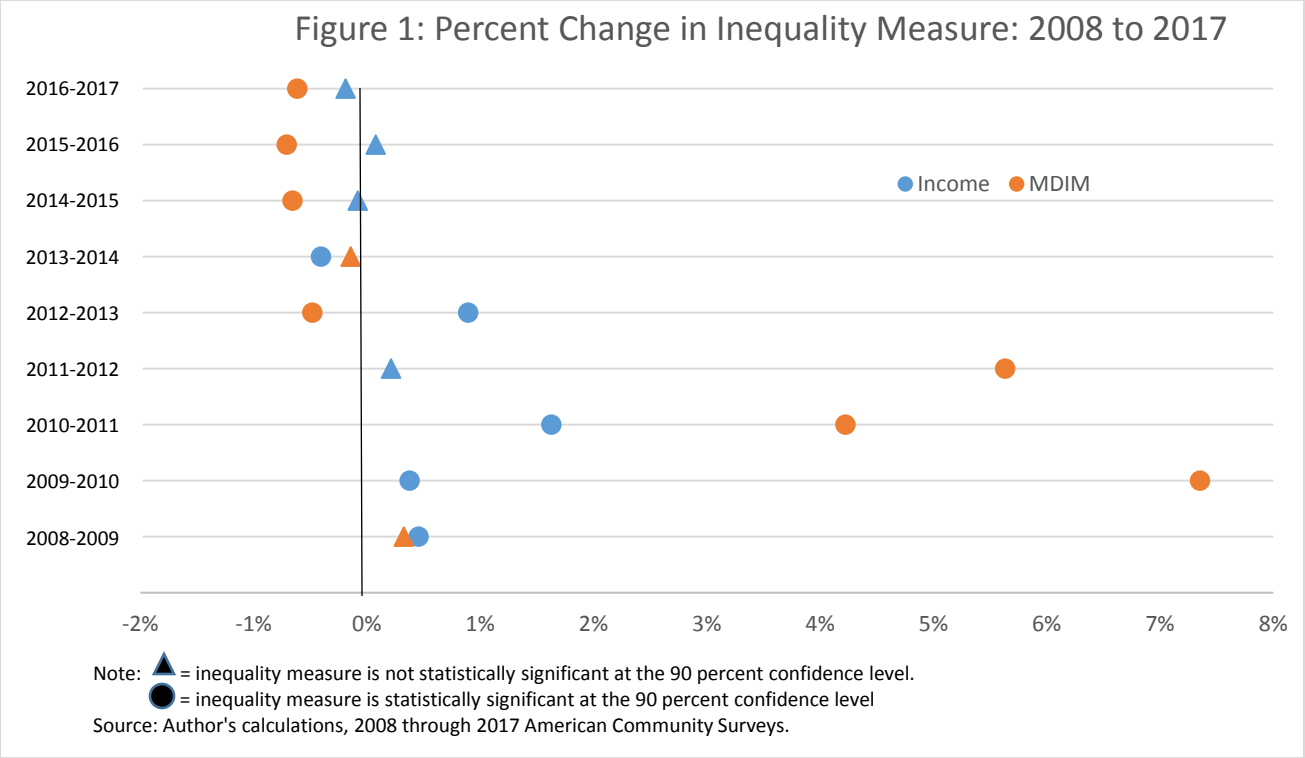
Table 9: Multidimensional Inequality using the Gini Index (ε = 2) - 2008 and 2017						
	2008		2017		Percent Change	
	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.
β = 1	0.1478	0.0002	0.1487	0.0001	*0.59	0.14
β = 0	0.1637	0.0001	0.1663	0.0001	*1.60	0.11
β = -1	0.2063	0.0002	0.2136	0.0002	*3.50	0.13
β = -2	0.2375	0.0002	0.2458	0.0002	*3.50	0.14

*** percent change in inequality measure is statistically significant at the 90 percent confidence level.**
Source: Author's calculations, 2008 and 2017 American Community Surveys.

Year-to-year changes in inequality

In this section of the paper, rather than just listing the overall change from 2008 to 2017, year-to-year changes in inequality are presented. In order to reduce the complexity of having so many different measures, a preferred specification is chosen for some figures and tables. The preferred specification is the multidimensional Atkinson Index with moderate inequality aversion (ε = 2) and moderate substitution of dimensions (β = -1). The multidimensional measure (MDIM) is also compared to income inequality (Atkinson Index with ε = 2).

In Figure 1, the percent change from year to year in income inequality and multidimensional inequality are shown. While Table 6 showed that income inequality increased and Table 8 showed that multidimensional inequality increased overall from 2008 to 2017, multidimensional inequality actually increased each year from 2009 to 2012 and then decreased each year from 2012 to 2017, with the sole exception of 2013-2014 in which the change was not statistically significant. The pattern for income inequality differs in a few ways. First, income inequality increased from 2008 to 2009 while the change in multidimensional inequality was not statistically significant. Conversely, multidimensional inequality increased for 2011 to 2012 while the change in income inequality was not statistically significant. Second, from 2012 to 2013, income inequality increased while multidimensional inequality decreased. Third, from 2013 to 2014 income inequality decreased while there was no statistically significant change in multidimensional inequality. Finally, each year from 2014 to 2017, multidimensional inequality decreased while changes in income inequality were not statistically significant.



State differences in inequality

In Table 10, a limited number of inequality measures are presented with varying amounts of inequality aversion and dimensional substitutability along with an income inequality measure. Rather than presenting the actual inequality measure, an index was calculated for each measure in which the state value is equal to 100 if it is equal to the U.S. inequality measure, less than 100 if the measure is less than U.S. inequality measure, and greater than 100 if the measure is greater than the U.S. inequality measure. This was done in order to more easily compare amongst measures.

Before discussing the overall results, remember that table 7 showed that some dimensions had higher inequality in New York and some dimensions had higher inequality in California, which meant that no definitive conclusions about which state had higher overall inequality could be made. In Table 10, each multidimensional inequality measure shows higher inequality for New York than for California. This shows the importance of a multidimensional measure in comparing inequality in different states.

There are two main takeaways from Table 10. The first is that the multidimensional measures were not consistently greater than or less than the U.S. measure for all states. In 36 states, all measures were less than or not significantly different than the U.S. measure; in 6 states and the District of Columbia all measures were greater than or not significantly different than the U.S. measure; and in 8 states some measures were greater than and some measures were less than the U.S. measure.

Table 10: Multidimensional Inequality Measures by State - 2017

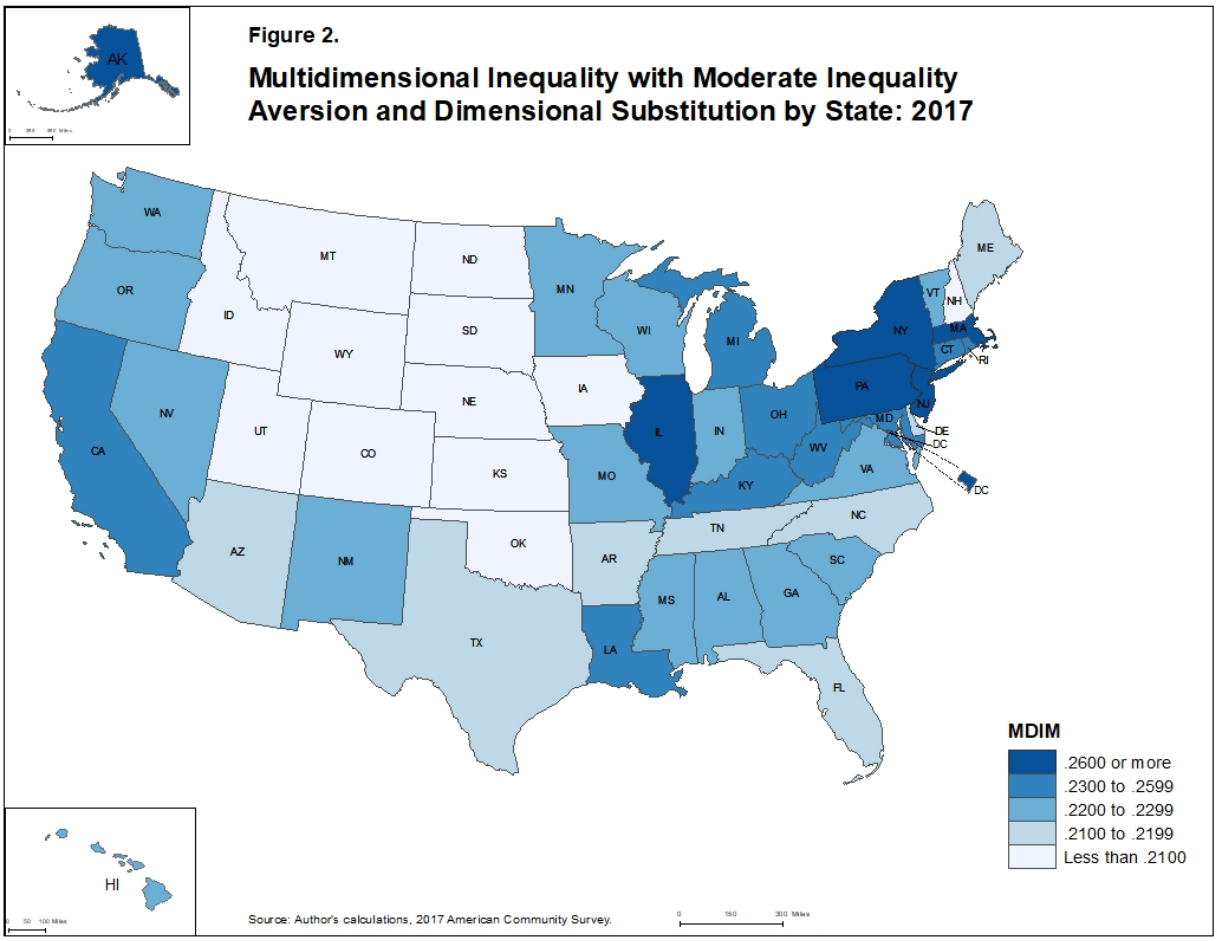
	Atkinson Index				Multidimensional Gini Index		Income Gini
Inequality aversion	Low	High	High	Moderate	Moderate	Moderate	Moderate
Dimension substitutability	Low	High	Low	Moderate	High	Low	N/A
	ε = 1, β = -2	ε = 3, β = 0	ε = 3, β = -2	ε = 2, β = -1	ε = 2, β = 0	ε = 2, β = -2	ε = 2
Alabama	98.78	*78.45	*93.91	*88.40	*95.27	*95.31	*98.74
Alaska	*118.90	93.63	*102.50	106.30	*103.30	102.50	*90.34
Arizona	102.20	*77.21	*92.27	*87.31	*97.00	*95.51	99.02
Arkansas	96.28	*76.76	*92.31	*86.44	*94.37	*95.20	*98.05
California	*116.80	*87.93	*94.49	*96.16	*105.60	*105.40	*105.90
Colorado	*92.69	*69.13	*91.36	*80.55	*90.96	*87.64	*95.39
Connecticut	*118.10	*86.34	*102.30	101.50	100.00	*93.77	*102.10
Delaware	93.81	*76.73	*93.91	*86.00	*93.35	*92.06	100.20
District of Columbia	*172.70	*212.00	*118.90	*167.20	*139.10	*160.80	*109.20
Florida	*96.09	*77.37	*91.97	*85.95	*94.98	*95.85	*102.90
Georgia	103.50	*79.93	*95.63	*90.84	*97.52	*95.27	*101.80
Hawaii	99.79	*82.80	*92.16	*89.30	*95.41	101.00	*92.13
Idaho	*82.67	*62.66	*84.05	*71.17	*86.71	*86.64	*92.55
Illinois	*117.10	100.20	*104.90	*109.30	*102.10	*103.30	100.60
Indiana	*94.88	*76.14	*95.84	*88.10	*91.09	*89.90	*92.09
Iowa	*90.34	*67.55	*93.24	*80.98	*86.99	*83.59	*89.77
Kansas	93.27	*67.81	*91.59	*80.77	*89.59	*86.09	*93.12
Kentucky	105.60	*85.12	*97.07	*94.81	*98.31	*98.60	*98.64
Louisiana	*106.80	*92.97	*98.61	100.30	99.55	*104.30	*101.30
Maine	96.34	*72.78	*94.76	*86.44	*90.04	*88.15	*93.17
Maryland	*114.20	*88.91	*103.70	*103.20	100.40	*95.02	*94.89
Massachusetts	*130.10	*106.70	*108.20	*117.40	*104.90	*104.00	100.60
Michigan	99.87	*81.64	*98.25	*93.39	*94.30	*93.28	*96.42
Minnesota	*106.80	*73.68	*98.79	*90.49	*89.85	*84.81	*92.20
Mississippi	101.30	*81.85	*93.19	*90.27	*96.87	99.59	99.02
Missouri	97.57	*75.75	*95.98	*88.48	*92.31	*90.02	*95.01
Montana	91.37	*68.63	*89.89	*79.04	*89.32	*88.38	*93.47
Nebraska	98.15	*68.22	*94.31	*83.47	*88.32	*83.90	*89.83
Nevada	102.00	*80.71	*95.17	*90.88	*97.61	*96.20	*97.69
New Hampshire	*84.39	*61.87	*89.61	*74.72	*85.06	*80.39	*89.67
New Jersey	*123.60	100.80	*105.50	*111.60	*104.70	*103.20	100.10
New Mexico	102.00	*80.88	*91.20	*88.04	*98.47	100.80	100.30
New York	*149.10	*180.40	*114.70	*150.40	*127.10	*148.70	*109.10
North Carolina	96.95	*74.85	*93.02	*85.75	*94.35	*92.47	*98.82
North Dakota	92.62	*69.18	*93.49	*82.20	*88.59	*85.18	*92.59
Ohio	*104.10	*82.82	100.30	*96.38	*94.08	*92.24	*95.05
Oklahoma	*92.78	*73.15	*90.45	*82.76	*93.68	*93.01	*97.32
Oregon	103.50	*80.85	*96.06	*91.58	*95.05	*95.53	*95.96
Pennsylvania	*119.80	101.10	*106.60	*112.60	*101.00	*101.30	*97.71
Rhode Island	*117.40	*88.05	101.50	102.20	100.20	*96.60	97.84
South Carolina	97.67	*78.65	*94.55	*88.73	*94.89	*94.63	100.50
South Dakota	92.62	*67.19	*92.01	*80.68	*88.65	*84.29	*92.15
Tennessee	95.83	*72.98	*91.87	*84.03	*93.74	*91.64	100.30
Texas	*103.00	*76.62	*90.85	*86.49	*98.80	*96.89	*102.10
Utah	*81.80	*59.55	*84.42	*70.42	*85.92	*83.04	*88.54
Vermont	100.60	*76.50	*95.89	*88.64	*92.48	*91.38	*93.61
Virginia	102.80	*75.01	*96.06	*88.86	*95.28	*89.71	*98.57
Washington	102.60	*77.01	*95.41	*88.93	*93.64	*92.21	*95.58
West Virginia	104.70	*89.52	*98.18	97.95	99.34	101.30	*96.20
Wisconsin	96.60	*73.49	*97.03	*87.85	*88.03	*85.87	*90.68
Wyoming	*81.30	*64.67	*87.07	*73.47	*86.73	*85.67	*89.99
United States	100.00	100.00	100.00	100.00	100.00	100.00	100.00

*** statistically significantly different from 100.00 at the 90 percent confidence level**
Source: Author's calculations, 2017 American Community Survey

The second is that there are significant differences between the multidimensional measures and the income measure. For example, compare the income Gini and the multidimensional Gini with β = -2.

In 34 states and the District of Columbia, the multidimensional Gini with β = -2 was consistent with the income Gini: both were lower than U.S. measures in 29 states, both were higher than the U.S measures in 3 states and the District of Columbia, and both were not significantly different than the U.S. measure in 2 states. However, in the other 16 states the multidimensional measure and the income measure were inconsistent. In 4 states, the multidimensional measure was lower and the income measure was higher than the U.S. measure while the opposite was true in 1 state; in 5 states, the multidimensional measure was lower than the U.S. measure and the income measure was not significantly different from the U.S. measure while the opposite was true in 3 states; and in 3 states the multidimensional measure was higher than the U.S. measure and the income measure was not significantly different from the U.S. measure.

For a visual representation of multidimensional inequality, the preferred specification from Figure 1 is shown for states in Figure 2. The states with the highest multidimensional inequality were bunched in the Northeast while the states with the lowest multidimensional inequality were bunched in the Northwest.



Using Figure 2 and the fact that U.S. multidimensional inequality using the specification from Figure 2 was .2509, the states can be broken up into three groups: states in which multidimensional inequality was greater than, less than, or not significantly different than U.S. multidimensional inequality. In Table 11, the characteristics of these three types of states are compared. The vast

majority of states had multidimensional inequality lower than U.S. multidimensional inequality but those states were significantly smaller in population size on average and had lower overall price levels than the states that had multidimensional inequality greater than U.S. multidimensional inequality.

States with multidimensional inequality higher than U.S. multidimensional inequality had higher unemployment rates, lower poverty rates, lower percent of people without internet access, and a lower percent of the population living in rural areas than states with multidimensional inequality lower than U.S. multidimensional inequality. Furthermore, states with multidimensional inequality higher than U.S. multidimensional inequality had larger proportions of Blacks, Asians, foreign born, people with a high school degree, and people with a college degree than states with multidimensional inequality lower than U.S. multidimensional inequality.

Table 11: Characteristics of States by Level of Multidimensional Inequality - 2017

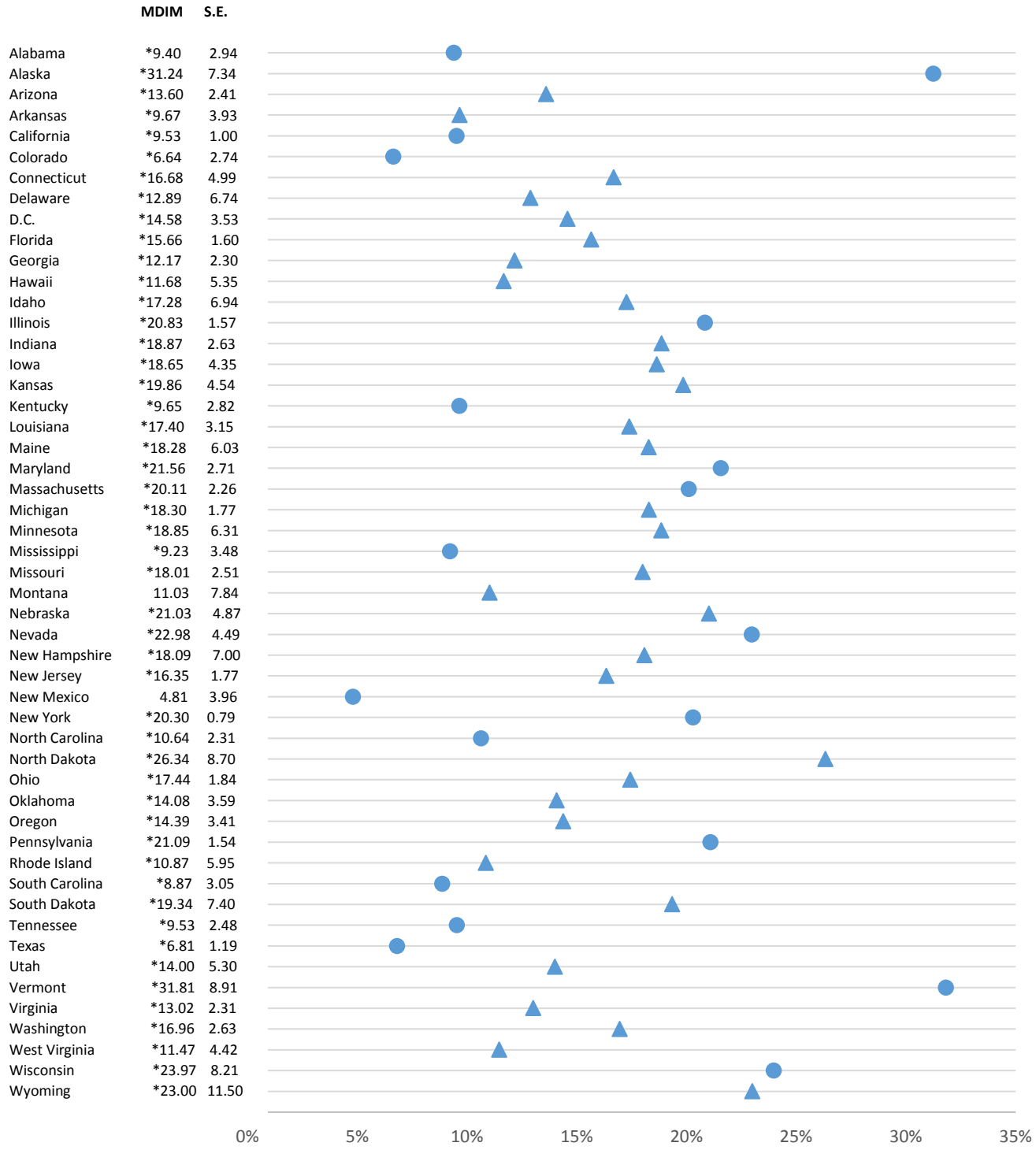
	Higher than U.S.		Lower than U.S.		Not significantly different than U.S.	
	Estimate	Std. Err.	Estimate	Std. Err.	Estimate	Std. Err.
Number of states	7	N/A	39	N/A	5	N/A
Average population	9,604,000	N/A	6,240,000	N/A	2,345,000	N/A
Average state price level[1]	107.6	N/A	98.0	N/A	97.3	N/A
Percent rural population	12.19	0.14	20.62	0.04	25.06	0.17
Unemployment rate	5.42	0.06	5.15	0.06	6.39	0.13
Poverty rate	12.13	0.11	13.62	0.05	15.29	0.21
Percent without internet access	8.74	0.25	9.57	0.23	11.37	0.40
Percent White	69.30	0.25	73.25	0.20	72.89	0.44
Percent Black	14.93	0.30	11.71	0.22	17.16	0.38
Percent Asian	6.81	0.01	5.40	0.03	2.95	0.03
Percent Hispanic	15.88	0.07	19.86	0.05	9.45	0.05
Percent foreign born	16.99	0.19	13.10	0.12	8.07	0.11
Percent without a high school degree	10.84	0.31	12.17	0.23	12.00	0.43
Percent with high school degree	27.44	0.24	26.65	0.18	32.20	0.23
Percent with some college education	24.83	0.11	30.18	0.08	26.54	0.21
Percent with college degree	36.89	0.45	31.00	0.33	29.21	0.43

Notes: [1] State price levels measured by Regional Price Parities published by the Bureau of Economic Analysis.
Three of the state characteristics have standard errors listed as N/A. This is done because these are not estimates from a survey.
Source: Author's calculations, 2017 American Community Survey.

Changes in State Level Multidimensional Inequality from 2008 to 2017

In Figure 3, the analysis of geography and time are combined in order to examine how state multidimensional inequality changed from 2008 to 2017. Once again the preferred specification with moderate inequality aversion and moderate dimensional substitutability was used. There are three main takeaways from this figure. First, multidimensional inequality increased in 48 states and the District of Columbia and did not change significantly in 2 states (Montana and New Mexico). Second, the amount of the increase varied significantly among the 48 states and D.C. Third, multidimensional inequality increased more than U.S. multidimensional inequality in 9 states and increased less than U.S. multidimensional inequality in 10 states. The increase in multidimensional inequality was not significantly different than the increase in U.S. multidimensional inequality in 31 states and the District of Columbia.

Figure 3: Percent Change in State Multdimensional Inequality Measure - 2008 to 2017



Note: △ = state estimate is not significantly different from U.S. estimate at the 90 percent confidence level.
○ = state estimate is significantly different from U.S. estimate at the 90 percent confidence level.
* = state estimate is statistically different from zero at the 90 percent confidence level.
Source: Author's calculations, 2008 and 2017 American Community Surveys.

Sensitivity Analysis

In the final three tables, different types of sensitivity analysis are performed. First, what happens to multidimensional inequality measures when different standardization and weighting methods are used? Second, what happens to multidimensional inequality when an individual dimension is removed? Third, what happens to multidimensional inequality when the health dimension is defined in different ways?

Decisions about the standardization and weighting methods were made at the beginning of the process. The divide-by-the-mean standardization method and the equal weighting method were chosen, though reasonable arguments can also be made for the subtract-the-minimum-and-divide-by-the-range standardization method and the data-driven weighting method.

Two points are readily apparent from the sensitivity analysis done in Table 12. First, the difference in estimates due to the weighting scheme is substantially less than the difference in estimates due to the standardization method. The mean standardization was chosen because the other standardization method was extremely sensitive to whatever the minimum value of the dimension was. This led to large year to year changes in multidimensional inequality due to extremely small changes in a dimension.

Second, there was not a consistent effect of either decision across multidimensional inequality measures. For the weighting scheme using the divide-by-mean standardization method, there was no significant effect on the first measure, a decrease in inequality for the second measure, an increase in inequality for the third and fourth measures, and a decrease in inequality for both Gini measures. Using the subtract-the-minimum-and-divide-by-range standardization method, all measures were higher using data driven weights. For the standardization method, there was an increase in inequality for the first measure and for the Gini measures and a decrease in inequality for the other three Atkinson measures. This result holds for both weighting methods.

Table 12: Alternative Methods - 2017

Standardization method	Weighting method		ε = 1, β = -2	ε = 3, β = 0	ε = 3, β = -2	ε = 2, β = -1	Gini, ε = 2, β = 0	Gini, ε = 2, β = -2
Divide by mean*	Equal*	Estimate	0.1821	0.1635	0.5744	0.2509	0.1663	0.2458
		Std. Err.	0.0010	0.0008	0.0004	0.0007	0.0001	0.0002
Divide by mean*	Data	Estimate	0.1835	0.1611	0.5890	0.2572	0.1555	0.2356
	driven	Std. Err.	0.0010	0.0008	0.0004	0.0007	0.0001	0.0002
Subtract min and	Equal*	Estimate	0.2443	0.1158	0.2579	0.1835	0.1812	0.2694
divide by range		Std. Err.	0.0012	0.0006	0.0010	0.0005	0.0001	0.0002
Subtract min and	Data	Estimate	0.2753	0.1231	0.2731	0.1993	0.1847	0.2777
divide by range	driven	Std. Err.	0.0012	0.0006	0.0009	0.0005	0.0001	0.0002

Note: * = method chosen for this paper
Source: Author's calculations, 2017 American Community Survey.

In Table 13, multidimensional inequality measures are presented which include all dimensions except for the one listed as the MDIM being without. How these measures compare to the MDIM is also included. There is no significant difference when the income dimension is excluded, but there are significant differences when the other dimensions are excluded. When vehicle ownership is excluded,

the inequality is significantly lower and when education, health, leisure, or housing are excluded, inequality is significantly higher.

Table 13: Effect of individual Dimensions on Multidimensional Inequality – 2017

	Estimate	Standard error	Difference from MDIM	Standard error
MDIM	0.2509	0.0007		
…without income dimension	0.2497	0.0013	-0.0012	0.0015
…without education dimension	0.2842	0.0014	*0.0333	0.0016
…without health dimension	0.3048	0.0014	*0.0539	0.0016
…without leisure dimension	0.2694	0.0012	*0.0185	0.0014
…without vehicle ownership dimension	0.1478	0.0007	*-0.1031	0.0010
…without housing dimension	0.2961	0.0013	*0.0452	0.0015

Note: * statistically significant at the 90 percent confidence level.
Source: Author's calculations, 2017 American Community Survey.

In Table 14, multidimensional inequality was calculated in three different ways. First, the MDIM is presented which used predicted health status to measure health. Second, the percent of people in the household with health insurance was used to measure health. Third, the total number of disabilities among all household members was used to measure health. There was little difference between the first and third calculations. This was not surprising as predicted health status was largely based on disabilities. There was a bigger difference between the first and second calculations but there are two caveats. First, is that the difference in estimates was still not very large. Second, a concern is that health insurance status may not capture the well-being of individuals as well as disabilities or predicted health status. For example, nearly all people age 65 and over had health insurance in 2017 (99.2 percent), but it is unlikely that they all would be considered healthy.

Table 14: Effect of Different Definitions of the Health Dimension on Multidimensional Inequality – 2017

	Estimate	Standard error	Difference from MDIM	Standard error
MDIM	0.2509	0.0007		
Health = Percent of household with health insurance	0.2717	0.0007	*0.0208	0.0010
Health = Number of disabilities in the household	0.2526	0.0007	*0.0017	0.0010

Note: * statistically significant at the 90 percent confidence level.
Source: Author's calculations, 2017 American Community Survey.

Conclusion

Inequality is increasingly being studied in a multidimensional setting. Income may be the most common dimension studied, but it is not the only important one. In the literature and in this paper, inequality is analyzed in two ways: individual measures and aggregated into a single measure.

Looking at individual measures is instructive about inequality in individual dimensions, but is generally not helpful in comparing overall inequality over time or between different geographical areas. For example, income, vehicle ownership, and housing inequality increased from 2008 to 2017 while education, health, and leisure inequality decreased. This difference doesn't allow us to say anything about what happened to overall inequality over this time period.

While we lose some information about individual dimensions when they are aggregated together, this method does allow us to determine differences in overall inequality over time and across geographies. For example, most of the overall inequality measures increased from 2008 to 2017, despite the differences over time in the individual dimensions.

It is also important not to generalize about changes in inequality from 2008 to 2017. While inequality was higher in 2017 than it was in 2008, the MDIM actually increased each year from 2009 to 2012 and then decreased each year from 2012 to 2017, with the exception of 2013-2014 in which the change was not statistically significant.

The same warning about generalization should also be heeded when it comes to state differences. Each of the multidimensional inequality measures examined in this paper were not consistently greater than or less than the U.S. measure for a particular state.

A final comment about the results in this paper is that they are sensitive to the choice of parameters (Table 8), the standardization method and weighting method (Table 12), and the dimensions included (Table 13).

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