Jackknife and Other Replication Methods with a Reduced Number of Replicates

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2018 Joint Statistical Meetings
July 30, 2018

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United States Census Bureau
U.S. Department of Commerce
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Goals of the Research

G1. Identify replication variance estimators that use a reduced set of replicates.

G2. Simple expression for the estimator of a variance.

G3. Appropriate for systematic random sampling from an ordered list – which we will refer to as *sys*.

Two parts

Part 1 – Single-stage sample design.

- Estimate the variance from a *sys* sample design.
- *sys* – defined as *systematic random sample from an ordered list*.

Part 2 – Two-stage sample design.

- Estimate the second-stage variance from a general first-stage sample design and a *sys* sample design in the second stage.

Part One: Single Stage sample Designs

$$\hat{Y} = \sum_{h} \sum_{k \in s_h} w_k \, y_k$$

$$\hat{v}\left(\hat{Y}\right)$$

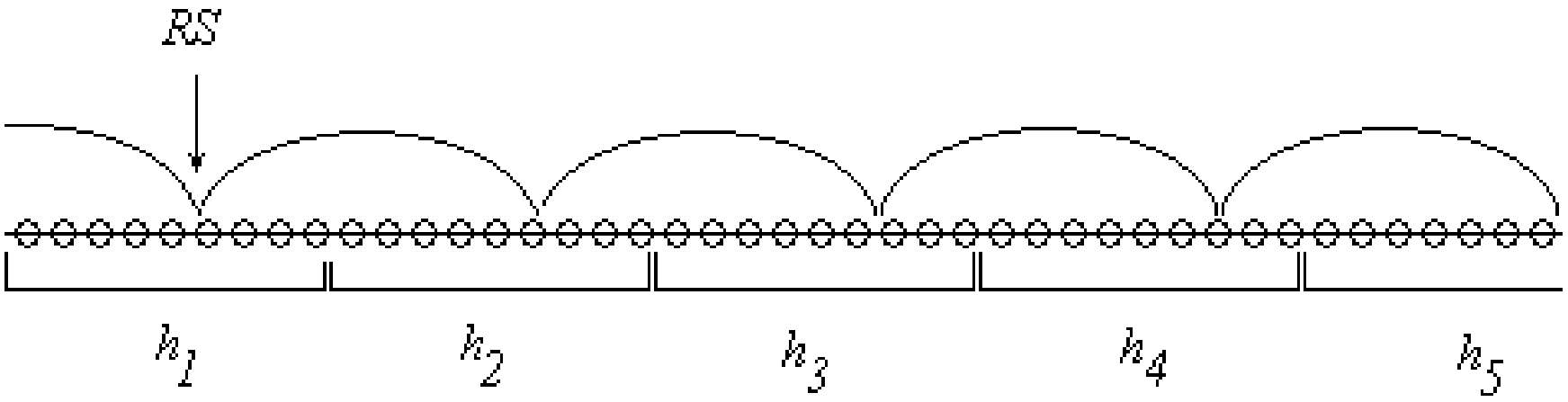
Part Two: Two Stage sample Designs

$$\hat{Y} = \sum_{h} \sum_{i \in s_h} \sum_{k \in s_i} w_i w_k y_k$$

$$\hat{v}\left(\hat{Y}\right) = \hat{v}_{\mathrm{PSU}}\left(\hat{Y}\right) + \hat{v}_{\mathrm{SSU}}\left(\hat{Y}\right)$$

$$\hat{v}_{\mathrm{SSU}}\left(\hat{Y}\right) = \sum_{h} \sum_{i \in s_h} \frac{\hat{v}\left(\hat{Y}_i\right)}{\pi_i^2}$$

Systematic Random Sampling from an Ordered List (*sys*)



-Can be treated as a cluster sample (Cochran 1977)
-Can be treated as a $n_h = 1$ – implicit stratification (Megill *et al*. 1987)

Part 1: Replication methods considered

- Jackknife replication -- Delete 1 unit (JK-1)
- Delete-a-Group Jackknife (DAGJK)
  - Kott (2001)
- Balanced Repeated Replication (BRR)
- Successive Difference Replication (SDR)
  - Fay and Train (1995) and Ash (2014)

Delete-a-Group Jackknife (DAGJK)

- Randomly assigns each sample unit to the R groups called D_r with d_r units in each group D_r.
- Used sys to form the groups.

$$\hat{v}_{JKDAG}(\hat{\theta}) = \frac{R-1}{R}\sum_{r=1}^{R}\left(\hat{\theta}_r - \bar{\hat{\theta}}_r\right)^2$$

$$F_r = \begin{cases} n/(n-d_r) & k \notin D_r \\ 0 & k \in D_r \end{cases}$$

Successive Difference Replication

- Mimics the Successive Difference estimator.
- The successive difference estimator is often good with the *sys* sample design. See Wolter (1984).
- Collapse strata estimator. (1,2), (2,3), (3,4), (4,5),…

$$\hat{v}_{\text{SDR–SSU}}(\hat{Y}) = \frac{4}{R}\sum_{r=1}^{R}(\hat{Y}_r - \hat{Y})^2$$

$$F_{k,r} = 1 + 2^{-\frac{3}{2}}a_{a_k,r} - 2^{-\frac{3}{2}}a_{b_k,r}$$

Part 1: Balanced Repeated Replication (BRR)

- Two sample units per stratum.
- Use BRR as SDR – a collapsed strata estimator
- (1,2), (3,4), (5,6),…

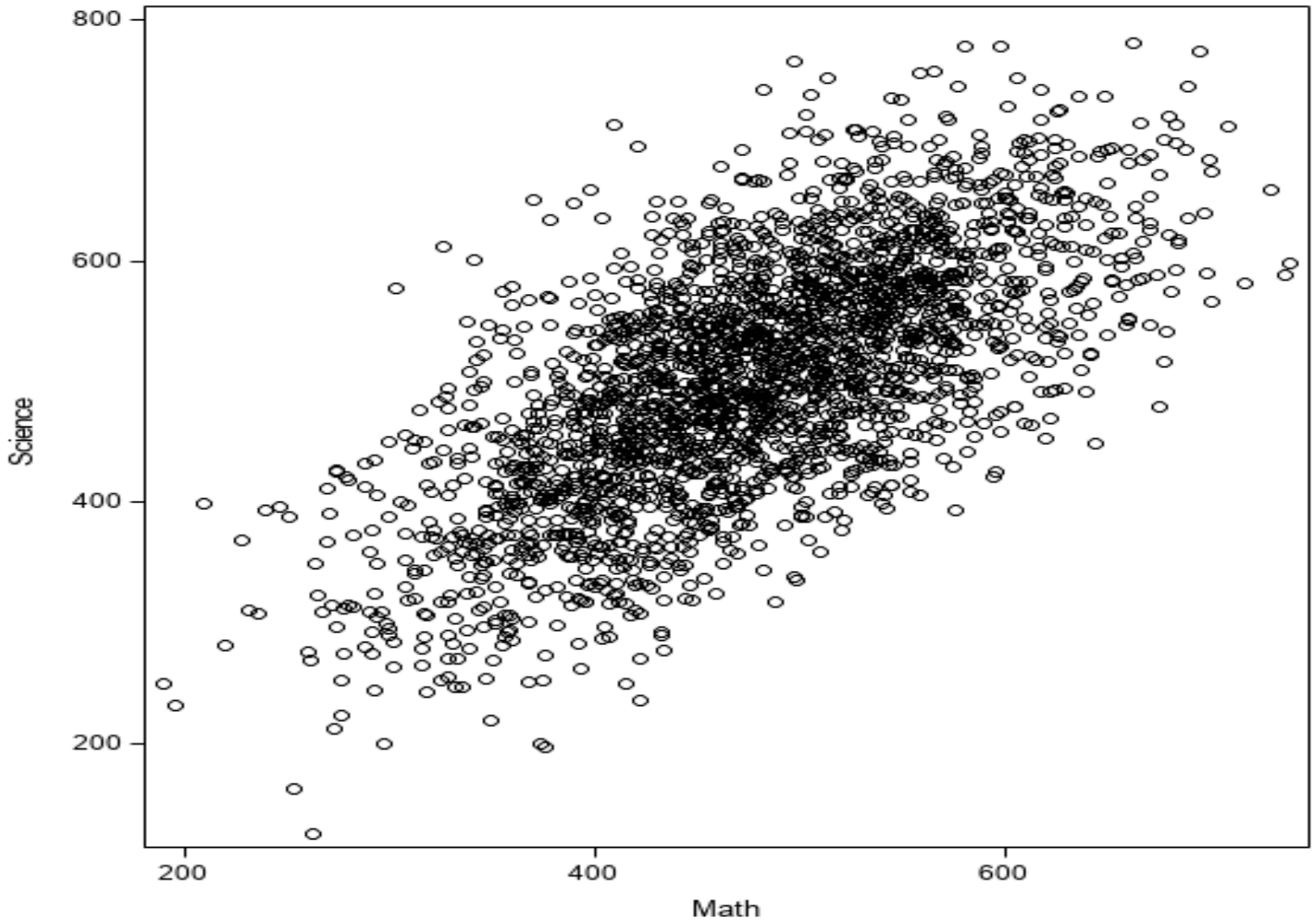
$$F_{r,h} = \begin{cases} 1 + (1-k)a_{r,h} & i = 1 \\ 1 - (1-k)a_{r,h} & i = 2 \end{cases}$$

$$\hat{v}_{\mathrm{BBR-FAY}}(\hat{Y}) = \frac{1}{R(1-k)^2} \sum_{r=1}^{R} \left(\hat{Y}_r - \hat{Y}\right)^2$$

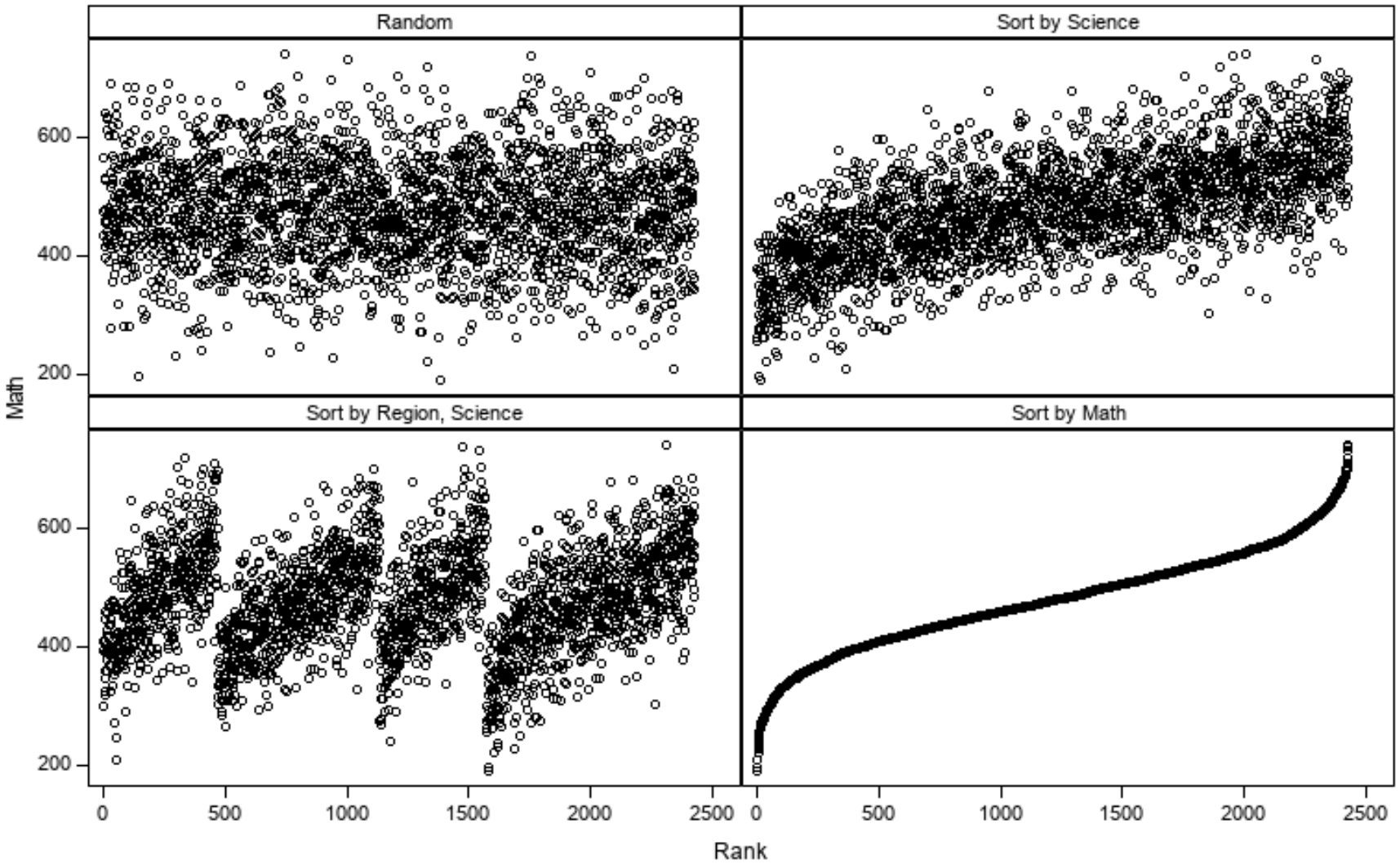
Empirical Example

- Used the complete population of 3rd Graders from Valliant, Dorfman, and Royall (2000).
- N = 2,427.
- Variable of interest total of math scores – y_k = math score
- Sort variables used
  - Science Scores
  - Region (4)

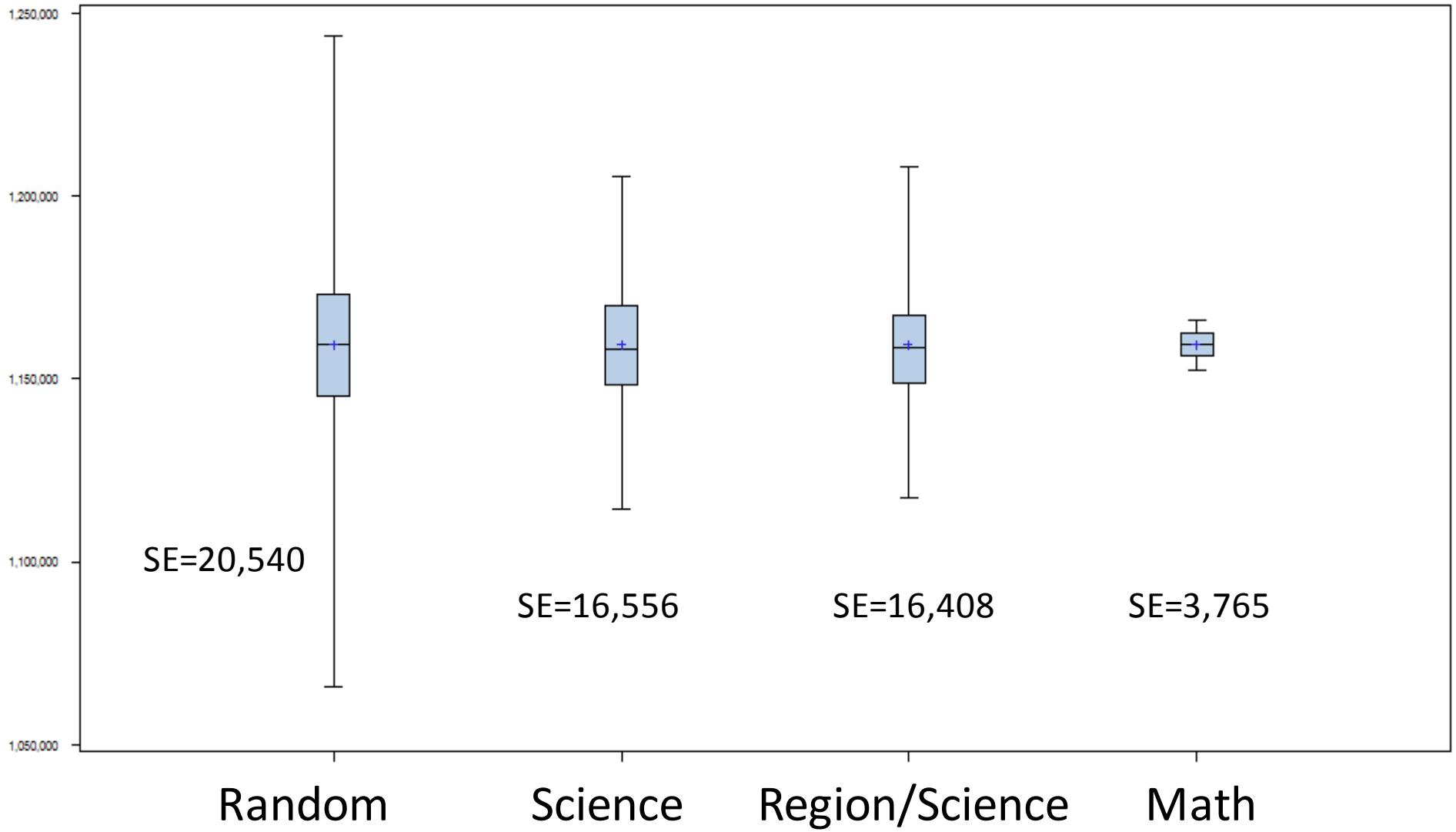
Math and Science Test Scorces



Sort Orders for *sys*

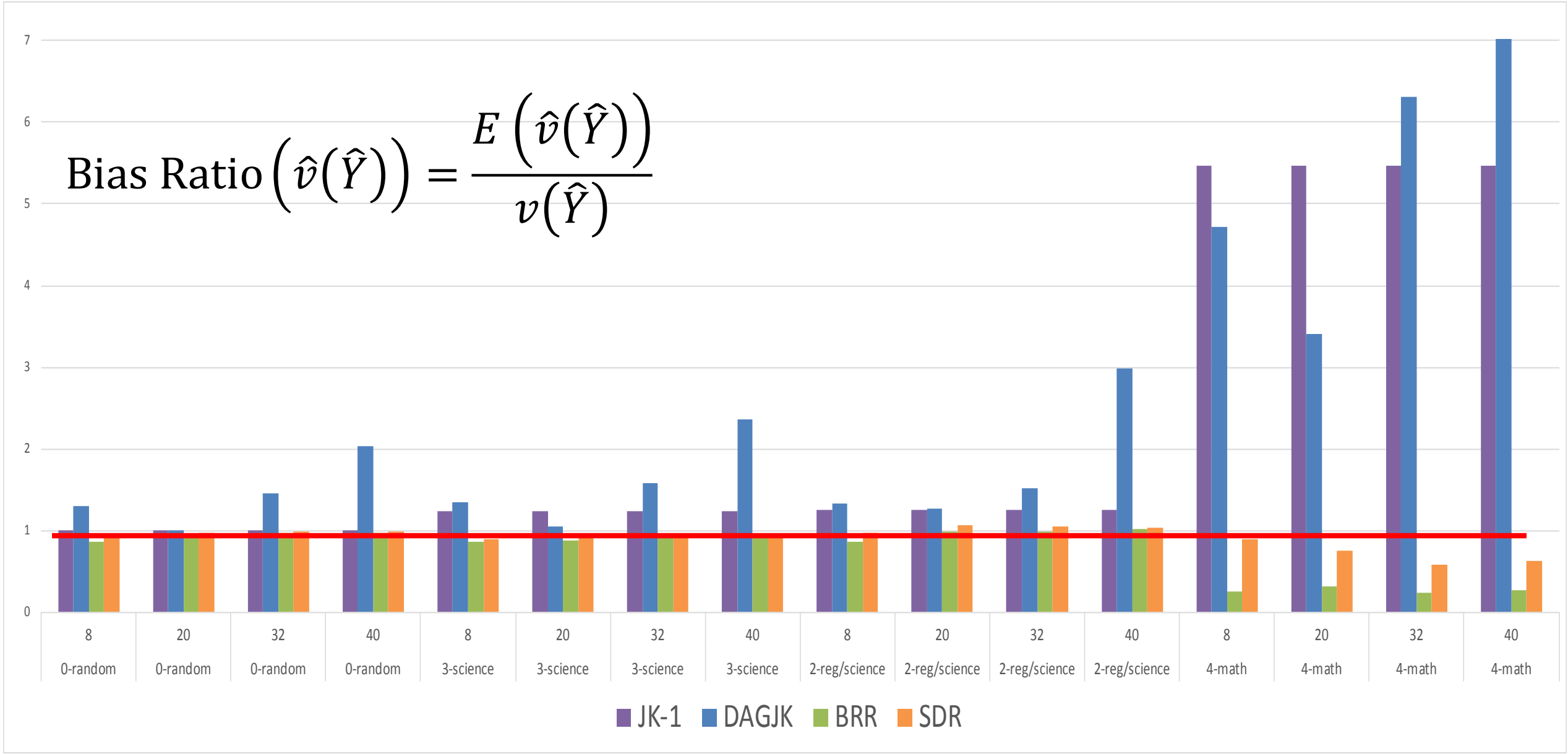


Actual Variances for Empirical Example



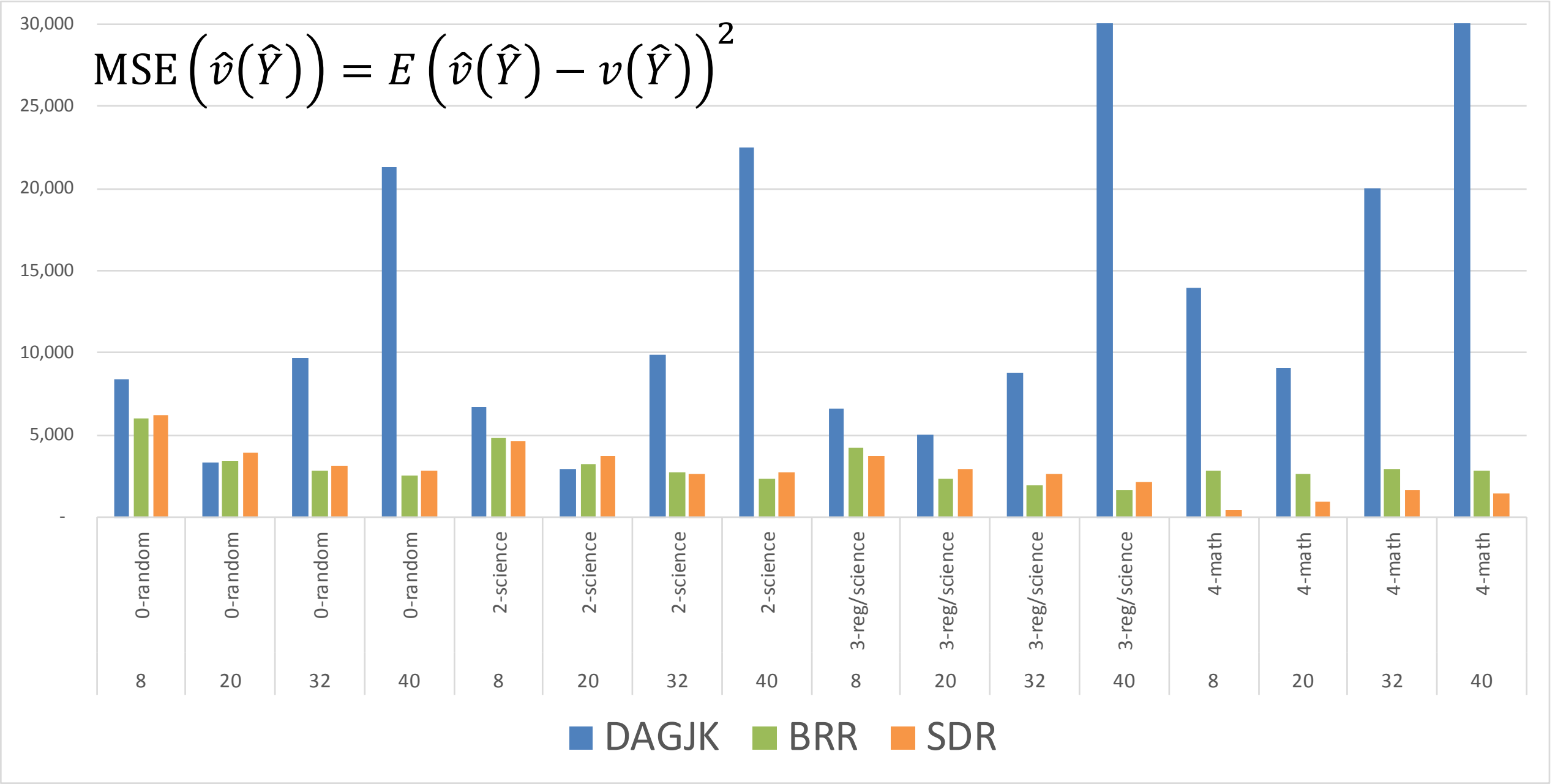
Bias Ratios

$$\text{Bias Ratio}\left(\hat{v}(\hat{Y})\right) = \frac{E\left(\hat{v}(\hat{Y})\right)}{v(\hat{Y})}$$

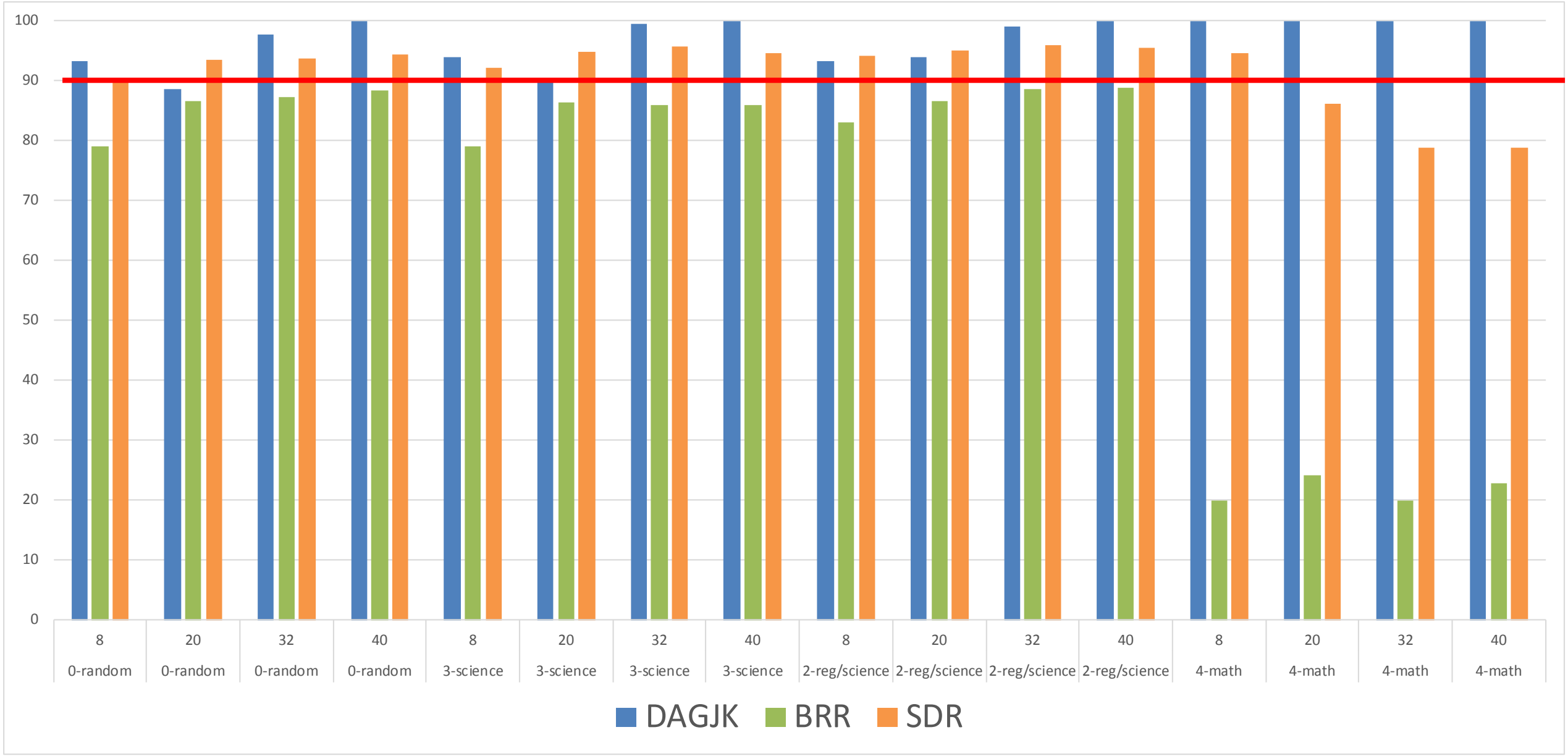


MSEs (as CVs)

$$\mathrm{MSE}\left(\hat{v}(\hat{Y})\right) = E\left(\hat{v}(\hat{Y}) - v(\hat{Y})\right)^2$$



90% Coverage Ratios



Part 1: Conclusions

C1. SDR is best for estimating the variance from a *sys* sample design.

C2. BRR was 2nd best.

C3. Leaving out a *sys* sample with the DAGJK does not do well at estimating the variance from a *sys* sample design.

Part 2: Replication methods for Second-Stage Variance

- Modified Successive Difference Replication (SDR2)
- Modified Balanced Repeated Replication (BRR2)
- Rizzo and Rust [2011] (RR)

$$v(\hat{Y}) = v_{\text{PSU}}(\hat{Y}) + v_{\text{SSU}}(\hat{Y})$$

We want an estimator for:

$$\hat{v}_{\text{SSU}}(\hat{Y}) = \sum_{h} \sum_{i \in s_h} \frac{\hat{v}(\hat{Y}_i)}{\pi_i^2}$$

Second-Stage Replicate Variance Estimation

- Modified Successive Difference Replication (SDR2)

$$F_{k,r} = 1 + \sqrt{1 - f_i}\left(2^{-\frac{3}{2}} h_{a_k,r} - 2^{-\frac{3}{2}} h_{b_k,r}\right)$$

- Modified Balanced Repeated Replication (BRR2)

$$F_{k,r} = \begin{cases} 1 + (1-k)a_{r,k} & k = 1 \\ 1 - (1-k)a_{r,k} & k = 2 \end{cases}$$

- Rizzo and Rust [2011] (RR)

$$F_{k,r} = \begin{cases} 1 + (1-k)a_{r,h} & j \in A_i \\ 1 - (1-k)a_{r,h} & j \in D_i \\ 1 & \text{otherwise} \end{cases}$$

Thanks

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