Seasonal Adjustment Subject to Frequency Aggregation Constraints

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July 31, 2019

Disclaimer

Introduction

- Presence of residual seasonality in published GDP figures
- Renewed interest in seasonality diagnostics and seasonal adjustment at the Bureau of Economic Analysis (BEA)
- Preliminary findings at BEA indicated that residual seasonality could occur as a result of aggregating monthly source data to quarterly frequency – Moulton (2016)
- Demonstration of phenomenon through simulations, theoretical models – McElroy (2016)

Indirect vs Direct Seasonal Adjustment

- Suppose we have (raw) monthly source data available, and we wish to obtain a quarterly seasonal adjustment

- **Indirect adjustment** – we seasonally adjust the monthly source data and aggregate the adjustment

- **Direct adjustment** – we aggregate the monthly source data and seasonally adjust the aggregate

- More control over outcomes with direct adjustment, so easier to ensure adequacy (i.e., the resulting adjustment does not exhibit seasonality) ...

Indirect vs Direct Seasonal Adjustment (2)

- But direct adjustment generally not equal to indirect adjustment (i.e., accounting relationships not preserved)

- If monthly seasonally adjusted numbers are published, then having quarterly numbers that do not satisfy this aggregation requirement is a drawback

- Further complication: sometimes, the monthly raw data is not available; i.e., the data at hand is a monthly seasonal adjustment, making it impossible to compute a direct adjustment

- That is, sometimes, indirect adjustment is the only option, and this adjustment may not necessarily be adequate

Seasonality in Frequency Aggregated Series

- "Frequency-aggregated seasonality" – when a change in sampling frequency via (flow) aggregation exhibits seasonality in the resulting aggregate
- E.g., we have a monthly time series that shows no seasonality; when aggregated to quarterly frequency, seasonality is observed
- Alternately, we have a monthly time series that is seasonal, is (adequately) seasonally adjusted, but seasonality is observed in the quarterly aggregate of the monthly adjustment
- Direct adjustment of the quarterly series would remove seasonality in either scenario, but then the direct adjustment will not equal the aggregate of the monthly adjustment.

Benchmarking Methodology

- Benchmarking problem: we have a time series sampled at a high and low frequency; for convenience, we can assume these are monthly and quarterly, respectively

- Literature on benchmarking is extensive, but issue of adequacy is not usually addressed

- Goal here is to try to adjust monthly and quarterly data such that both sets of seasonal adjustments are adequate

Some Notation

- Let the monthly series be denoted $\{X_{t,m}\}$ and its quarterly counterpart $\{X_{i,q}\}$, where $t = 3i + j$ for $j = 1, 2, 3$, where the data satisfy the following frequency aggregation property for quarter i:

$$X_{i,q} = X_{3i+1,m} + X_{3i+2,m} + X_{3i+3,m} \tag{1}$$

- Denote direct adjustments with N, so $\{N_{t,m}\}$ and $\{N_{i,q}\}$ – these may, but generally will not, satisfy the aggregation property above

- If not, we want modifications $\{Y_{t,m}\}$ and $\{Y_{i,q}\}$ that do preserve this property, are close to the direct adjustments, and are adequate

- If $\{N_{t,m}\}$ is available, but not $\{N_{i,q}\}$, then define $\{N_{i,q}\}$ as follows:
  - Aggregate $\{N_{t,m}\}$, test aggregate for seasonality
  - If adequate, done; else, seasonally adjust and use resulting adjustment as $\{N_{i,q}\}$

Some Notation (2)

- To minimize discrepancy between $\{Y_{t,m}\}$ and $\{N_{t,m}\}$, and between $\{Y_{i,q}\}$ and $\{N_{i,q}\}$, while preserving the frequency aggregation property and ensuring adequacy of both $\{Y_{t,m}\}$ and $\{Y_{i,q}\}$, amounts to minimizing the following expression for each quarter i:

$$\left(N_{i,q} - \sum_{j=1}^{3} Y_{3i+j,m}\right)^2 / N_{i,q} + \sum_{j=1}^{3} \left(N_{3i+j,m} - Y_{3i+j,m}\right)^2 / N_{3i+j,m}.$$

- Adequacy checked by applying some diagnostic δ to candidate solutions for monthly and quarterly series, compared against some threshold α

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What Diagnostic δ?

- McElroy (2018) used the QS diagnostic of Maravall (2012) as the diagnostic δ
- Concerns stemming from spurious detections of seasonality
- Instead, we use root diagnostic of McElroy (2019), which offers p-value for rejection of null hypothesis that seasonality is present to a given degree

Seasonality Diagnostic Based on Autoregressive Roots (McElroy, 2019)

- A causal invertible ARMA(p, q) process with MA polynomial $\theta(z)$ and AR polynomial $\phi(z)$ can also be represented using an MA(∞) $\psi(z) = \theta(z)/\phi(z)$

- A process is said to have "ρ-persistent seasonality of frequency $\omega \in [-\pi, \pi]$ (where $\rho \in (0, 1]$) iff its causal representation has coefficients $\{\psi_j\}$ with a ρ-persistent oscillatory effect of frequency $\omega \in [-\pi, \pi]$, such that $\pi(\rho^{-1} e^{i\omega}) = 0$, where $\pi(z) = 1/\psi(z)$"

- What is tested: for any given ω, the null hypothesis is

$$H_0(\rho_0) : \pi(r^{-1} e^{i\omega}) = 0 \quad \text{has solution } r = \rho_0$$

- The test statistic of $H_0(\rho_0)$ for a sample of size T is

$$T \left| \widehat{\pi}(\rho_0^{-1} e^{i\omega}) \right|^2$$

... In a Simpler Setting

- Simplifying, suppose we have an AR(p) process with AR polynomial $\phi(z)$, then $\pi(z)$ in the previous expressions is replaced by $\phi(z)$

- The null hypothesis says that for a given frequency ω, there is a root for the AR polynomial $\phi(r^{-1}e^{i\omega})$ at $r = \rho_0$

- If the magnitude of the AR polynomial (or the estimated AR polynomial) evaluated at $\rho_0^{-1}e^{i\omega}$ is large, that will produce a large test statistic; i.e., it would suggest that seasonality of a degree ρ_0 is not present in the tested process

- Note that this hypothesis test is laid out opposite – the null is positing the presence of seasonality to a certain degree, instead of no seasonality

Using Root Diagnostic

- Using the diagnostic, we look at p-values as a function of seasonal persistance ρ at frequencies $2\pi/4$ (for quarterly) and $2\pi j/12$ for $j = 1, 2, \ldots, 5$ (for monthly)

- Require

$$\max_{\rho \in (0.98,1)} p(\rho) \leq \alpha,$$

  where $p(\rho)$ denotes p-value as function of ρ determined by H_0

- Above says null hypothesis of seasonality of degree ρ can be rejected at level α for all $\rho \in (0.98, 1)$ – 0.98 corresponds to substantial degree of oscillation in autocorrelation function; lowering this value requires even weaker forms of seasonality be weeded out

- Notation-wise,

$$\delta\{Y_{1,m}, \ldots, Y_{3i+3,m}\} \leq \alpha, \quad \delta\{Y_{1,q}, \ldots, Y_{i,q}\} \leq \alpha,$$

  where δ indicates the maximum of p-values (3) computed on either monthly or quarterly data

Constrained Minimization

- Goal: minimize

$$\left(N_{i,q} - \sum_{j=1}^{3} Y_{3i+j,m}\right)^2 / N_{i,q} + \sum_{j=1}^{3} (N_{3i+j,m} - Y_{3i+j,m})^2 / N_{3i+j,m}.$$

subject to the constraints

$$\delta\{Y_{1,m}, \ldots, Y_{3i+3,m}\} \leq \alpha, \quad \delta\{Y_{1,q}, \ldots, Y_{i,q}\} \leq \alpha,$$

- Doable with Lagrangian techniques with inequality constraints or slack variables

- What we try: Convert constrained minimization problem into penalized minimization, iteratively increase the penalty until a solution has been achieved

Penalized Minimization

- I.e., introduce tuning parameters $\omega_m, \omega_q \geq 0$, and minimize objective function

$$\left(N_{i,q} - \sum_{j=1}^{3} Y_{3i+j,m}\right)^2 / N_{i,q} + \sum_{j=1}^{3} \left(N_{3i+j,m} - Y_{3i+j,m}\right)^2 / N_{3i+j,m}$$
$$+ \omega_m \left(\text{min}\left[\alpha - \delta\{Y_{1,m}, \ldots, Y_{3i+3,m}\}, 0\right]\right)^2$$
$$+ \omega_q \left(\text{min}\left[\alpha - \delta\{Y_{1,q}, \ldots, Y_{i,q}\}, 0\right]\right)^2$$

- Penalty terms are zero iff $\delta \leq \alpha$; solutions where $\delta > \alpha$ tend to be rejected

- Possible for adequate solutions to be obtained where $\omega_m = \omega_q = 0$, so using 0 as initial value is not unreasonable

- If candidate solution at initial values of ω_m and ω_q is not adequate at either frequency, increment both; repeat until adequate solution is achieved

General Framework

- Aggregate monthly series to quarterly series
- Use root diagnostic to determine whether either series is seasonal
- Construct indirect quarterly adjustment by aggregating monthly seasonal adjustment (or monthly raw series if deemed nonseasonal by root diagnostic)
- Use root diagnostic to determine whether monthly adjusted (or raw) series or indirect quarterly adjustment is seasonal
- If not, done; else, initialize ω_m and ω_q and begin nonlinear optimization of objective function
- Apply root diagnostic to reconciled series – if adequate, done; else, increment ω_m, ω_q and repeat

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Sample Applications

- Around 50 monthly economic series taken from some surveys conducted by U.S. Census Bureau, measuring quantities like shipments, construction spending, imports/exports

- Using $\rho \in (0.98, 1)$ calculated in 0.001 increments, majority of these series are such that null hypothesis of seasonality of degree ρ is rejected at both monthly and aggregated quarterly levels at an α of 0.1

- Some series (approx. 20–25%) where the raw monthly series is borderline seasonal (or borderline nonseasonal, and thus might be left as is), while the resulting quarterly aggregate is more noticeably seasonal

- Optimization starts with an initial value of $\omega_m = \omega_q = 0$, incrementing each by 1000 should a solution fail to satisfy the adequacy conditions

- Optimization is the major bottleneck; examples use the Bound Optimization by Quadratic Approximation (BOBYQA) algorithm of Powell (2009), as implemented in the `minqa` package in R

Example 1: Import Series

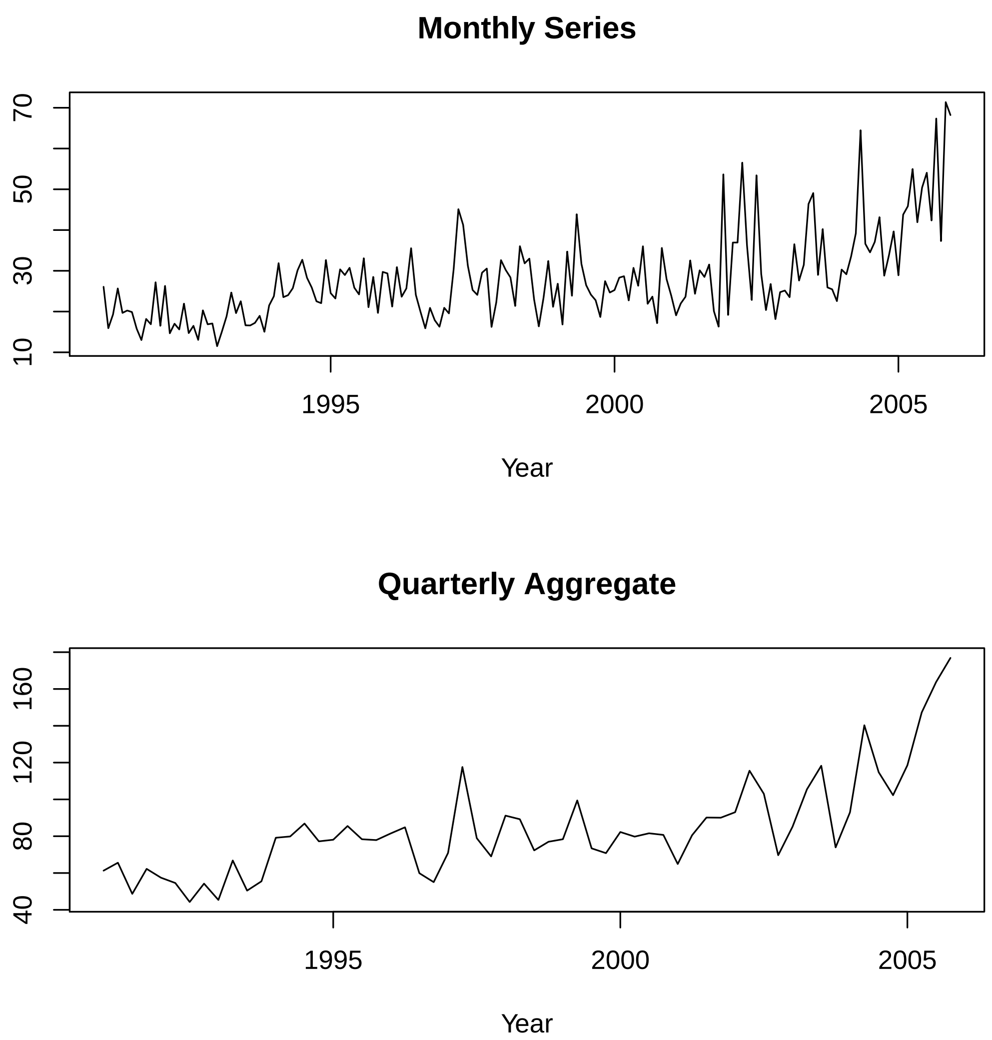

Figure 1: Monthly and quarterly aggregated series.

Example 1: Monthly

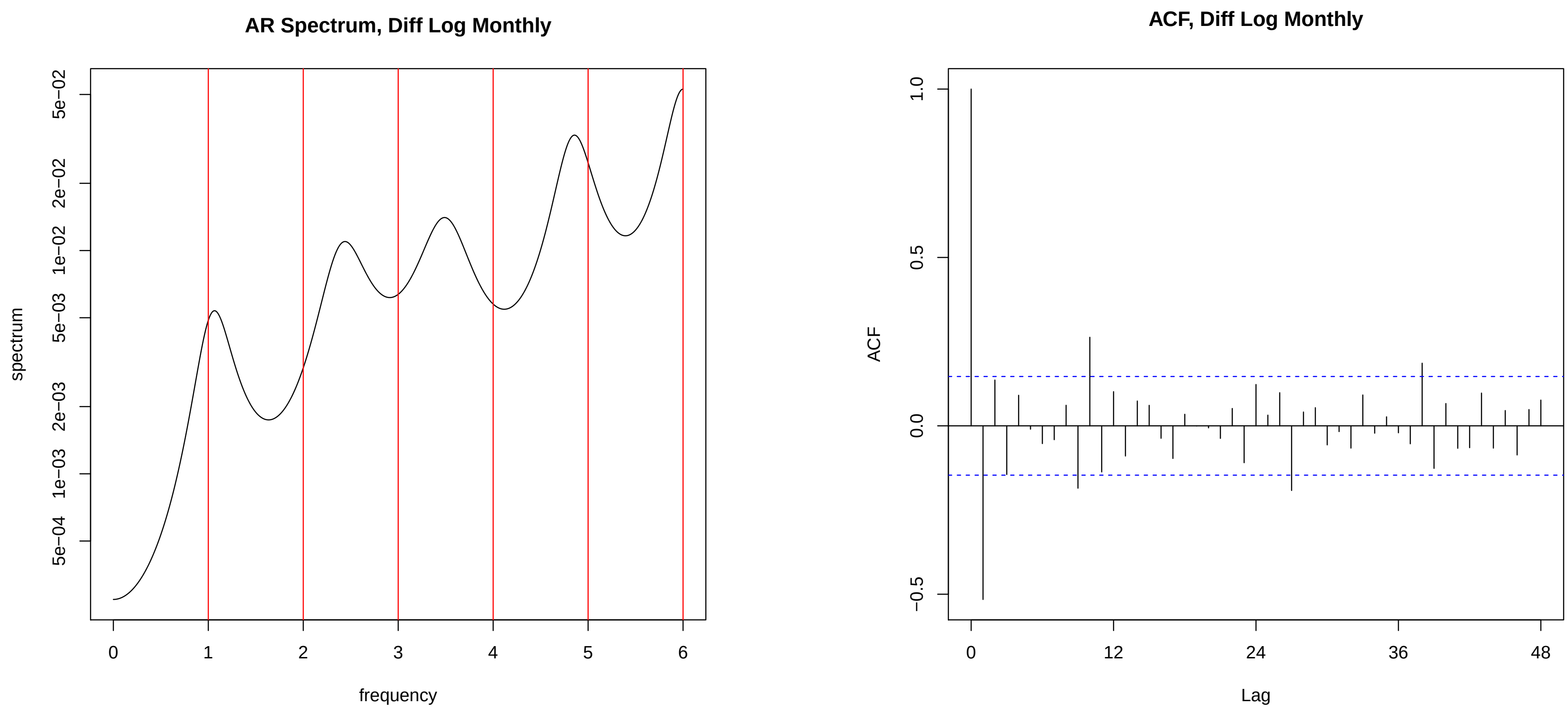

Figure 2: Autoregressive spectrum and autocorrelation function of the differenced log monthly series.

Example 1: Quarterly Aggregate

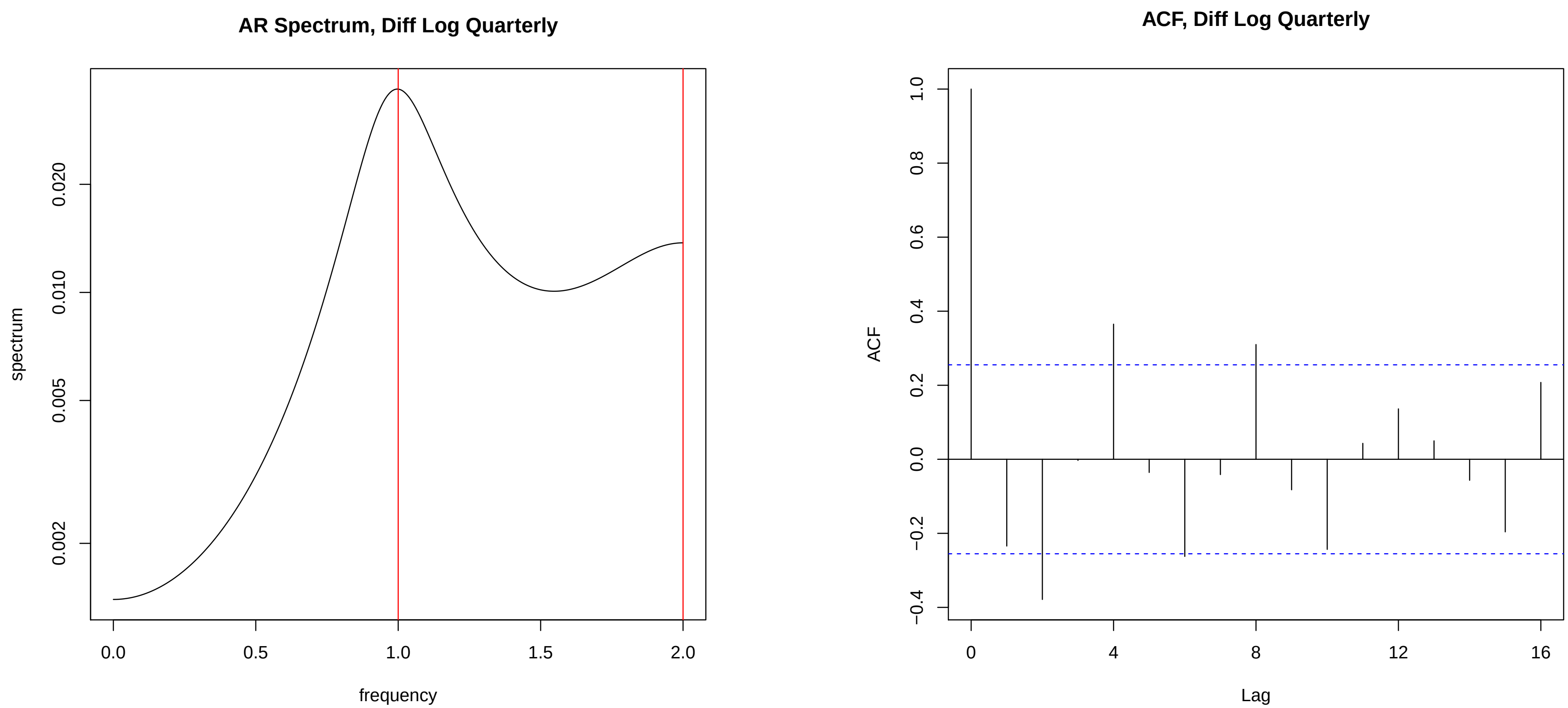

Figure 3: Autoregressive spectrum and autocorrelation function of the differenced log quarterly aggregate.

Example 1: Comments

- AR spectrum for monthly has peaks that do not align with monthly seasonal frequencies (red lines); AR spectrum for quarterly has a peak that appears to align with quarterly seasonal frequency (red lines)

- ACF for monthly series does not appear to show significant autocorrelations at seasonal lags; ACF for quarterly series appears to show significant autocorrelations at first and second seasonal lags

- That is, monthly series does not appear to be seasonal (or at least, not noticeably so), but aggregating suggests seasonality is present at a quarterly frequency

- Table 1 shows values of ρ for which the specified series is deemed seasonal using the root diagnostic (i.e., the series exhibits ρ-persistent seasonality); since monthly series was not adjusted, monthly and monthly SA should be identical, and quarterly aggregate and indirect quarterly seasonal adjustment values should be similar

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Series	ρ
Monthly	$\emptyset$
Qtrly Agg	$[0.980, 0.998]$
Monthly SA	$\emptyset$
Indirect Qtrly SA	$[0.980, 0.999]$
Direct Qtrly SA	$\emptyset$
Reconciled Mthly	$\emptyset$
Reconciled Qtrly	$\emptyset$

Table 1: Values of ρ for which the root diagnostic applied to the given series has a p-value exceeding $\alpha = 0.1$.

Example 1: Reconciled Monthly

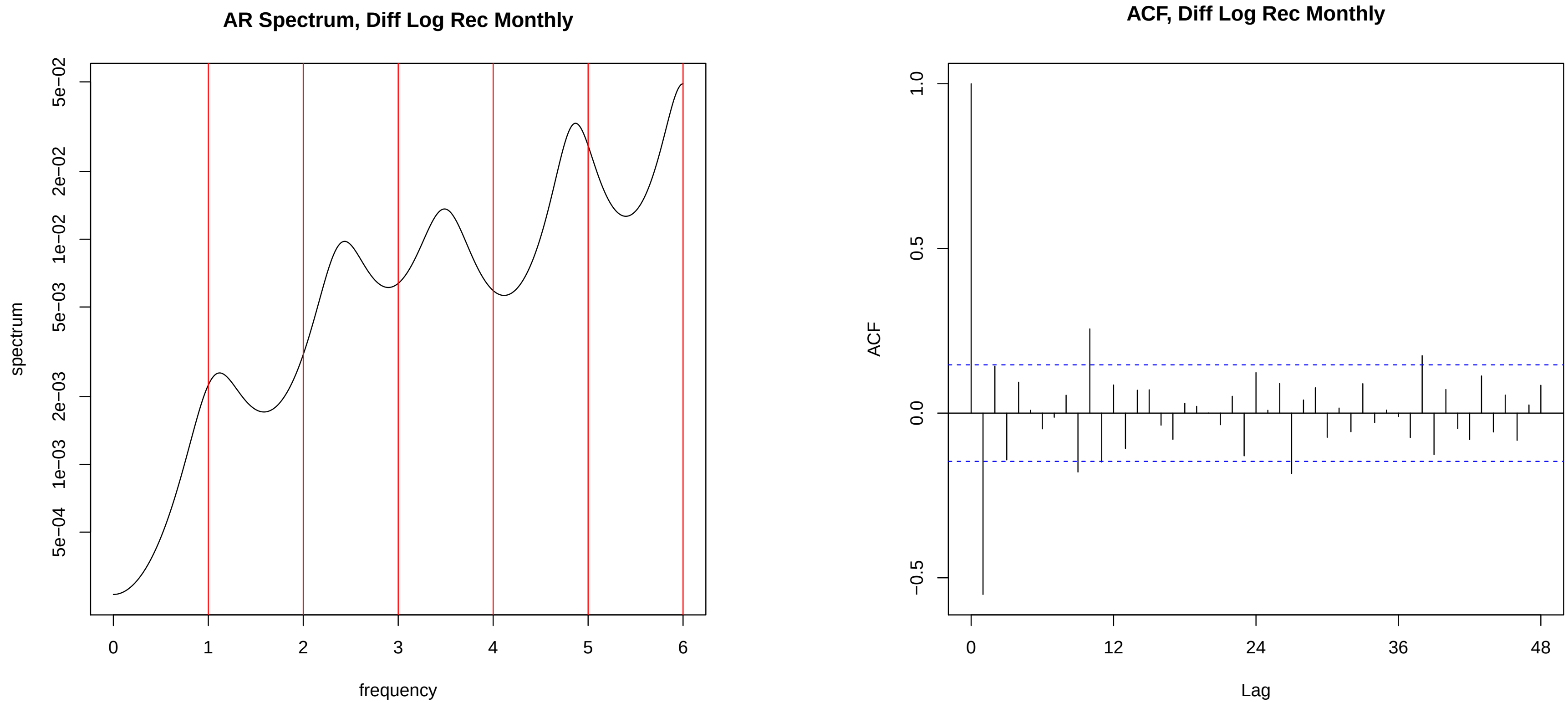

Figure 4: Autoregressive spectrum and autocorrelation function of the differenced log reconciled monthly series.

Example 1: Reconciled Quarterly

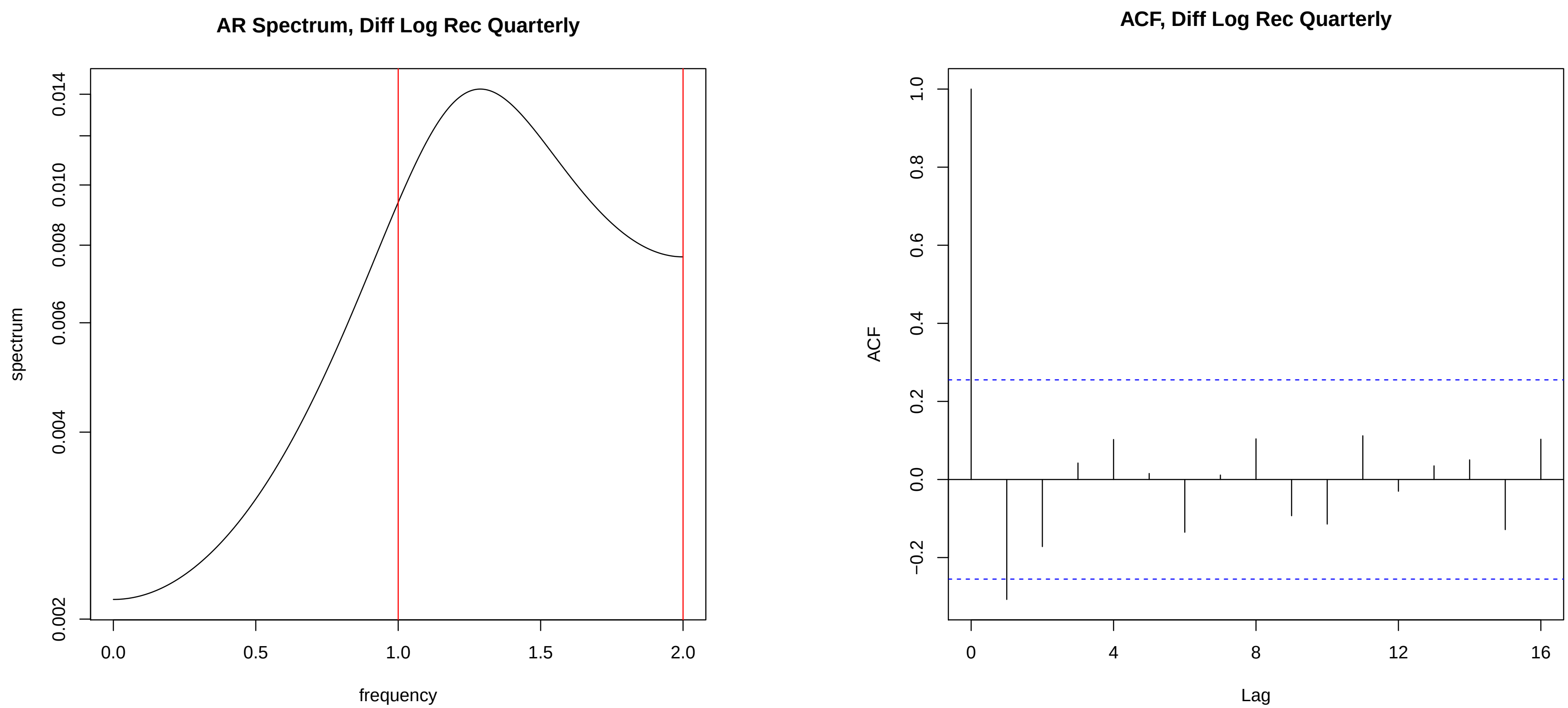

Figure 5: Autoregressive spectrum and autocorrelation function of the differenced log reconciled quarterly series.

Example 1: Monthly and Reconciled Monthly

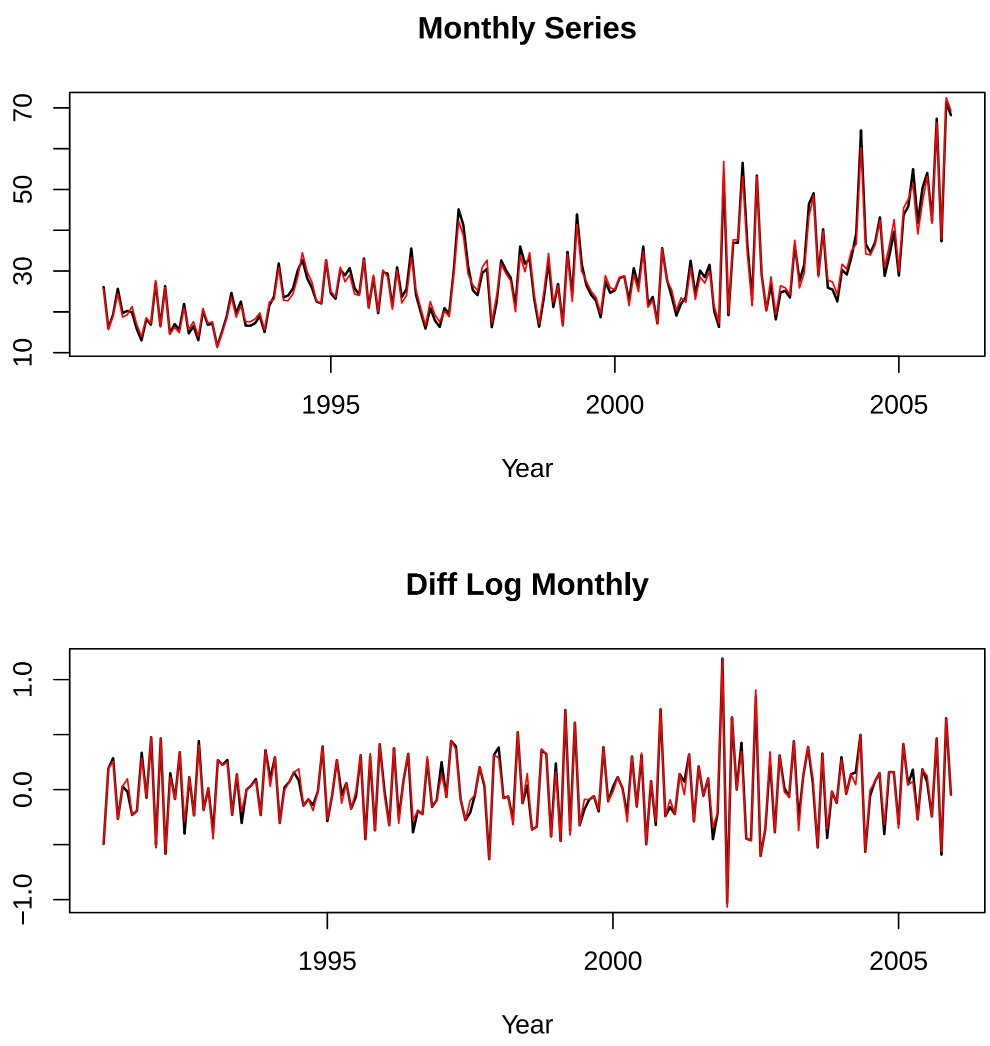

Figure 6: Monthly (black) and reconciled (red) series.

Example 1: Quarterly Aggregate and Reconciled Quarterly

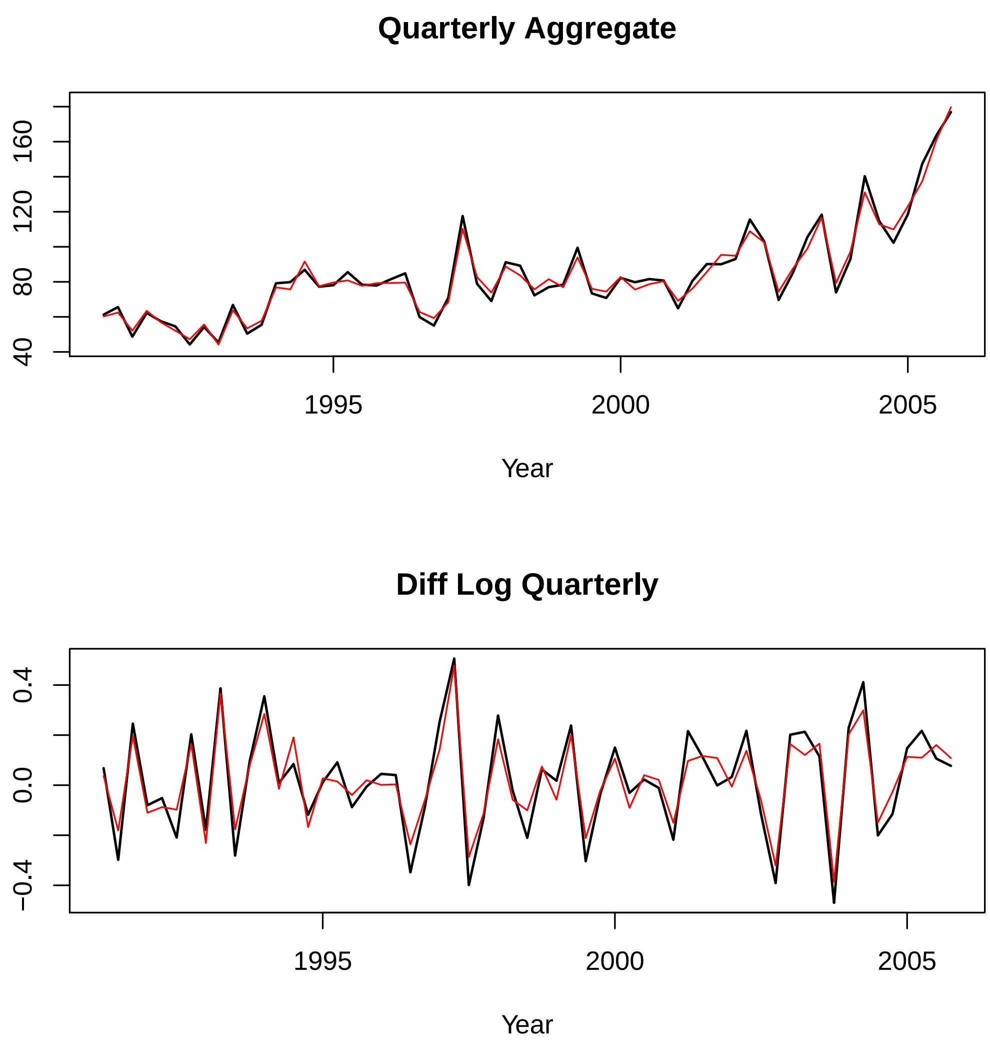

Figure 7: Quarterly aggregated (black) and reconciled (red) series.

Example 1: ... and Direct Quarterly Adjustment

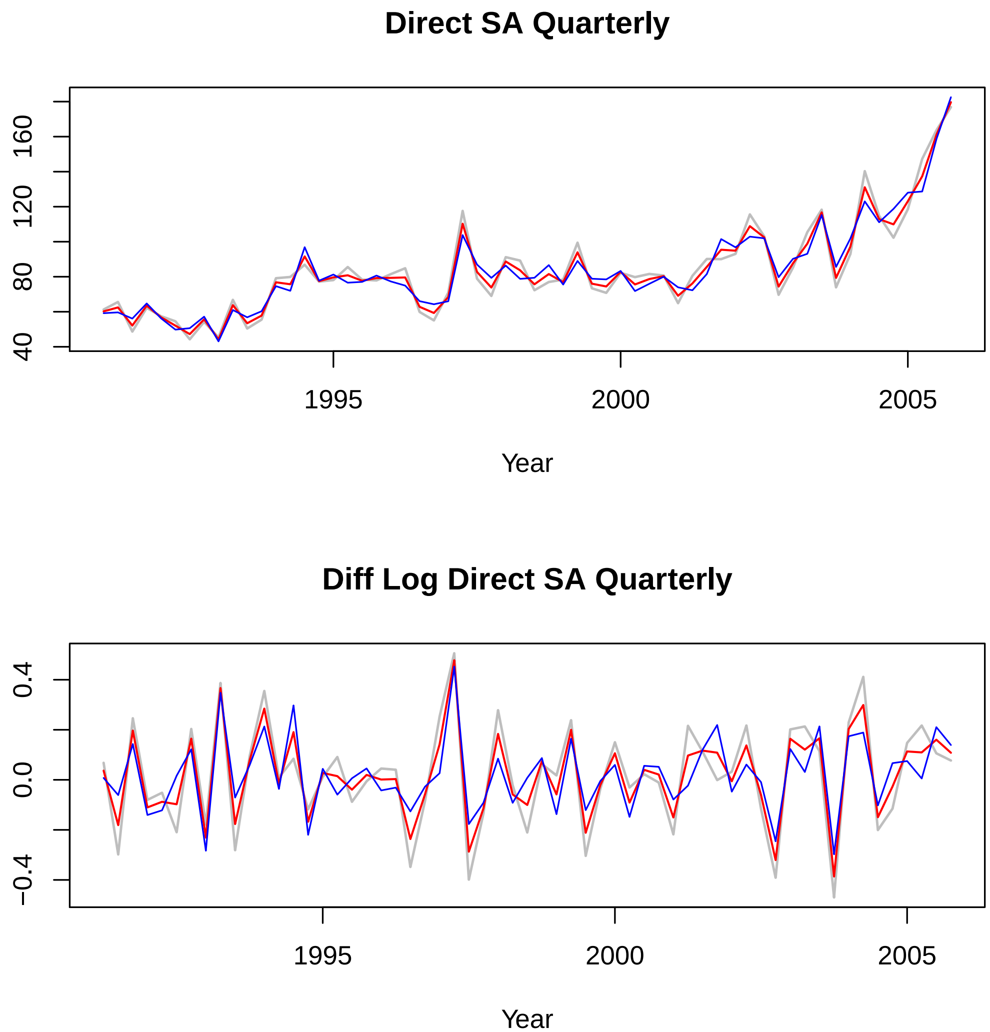

Figure 8: Quarterly aggregated (gray), reconciled (red), and directly adjusted quarterly (blue) series.

Example 2: Construction Spending Series

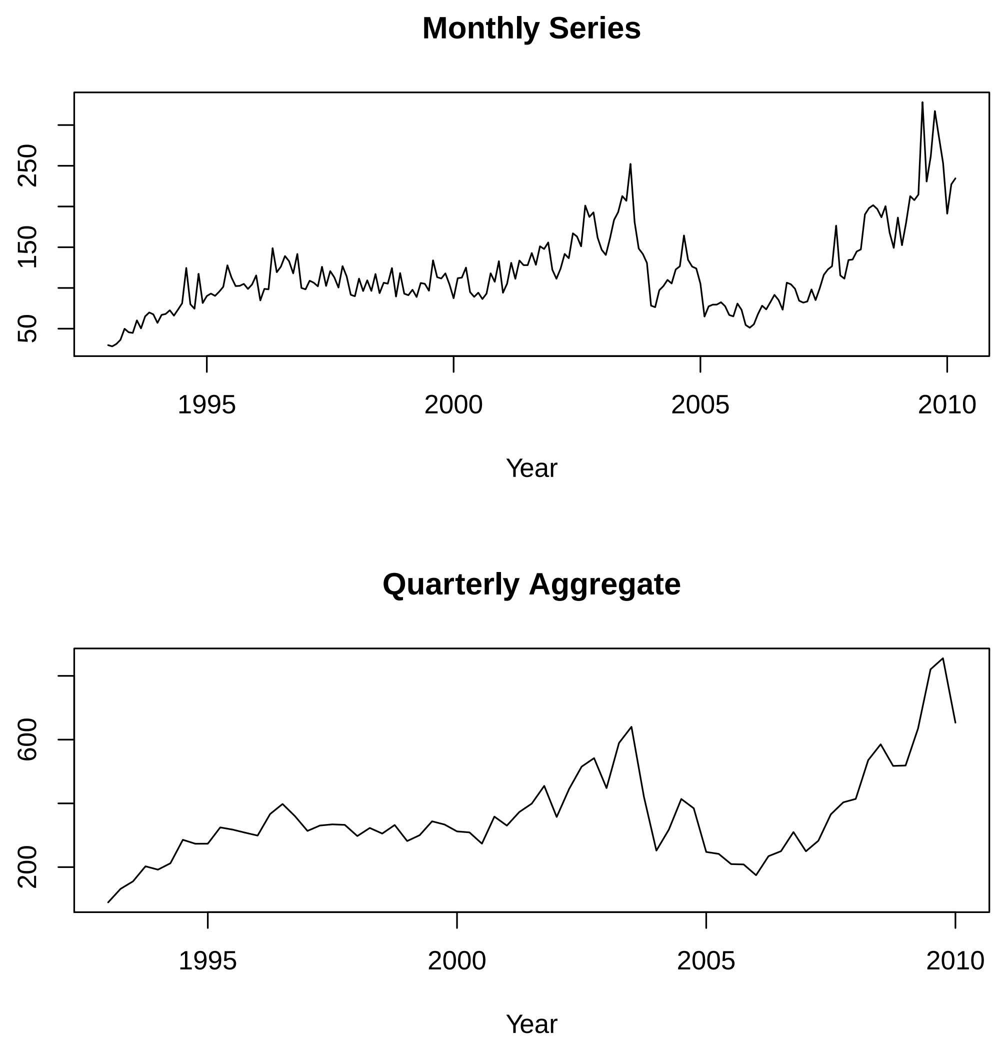

Figure 9: Monthly and quarterly aggregated series.

Example 2: Monthly

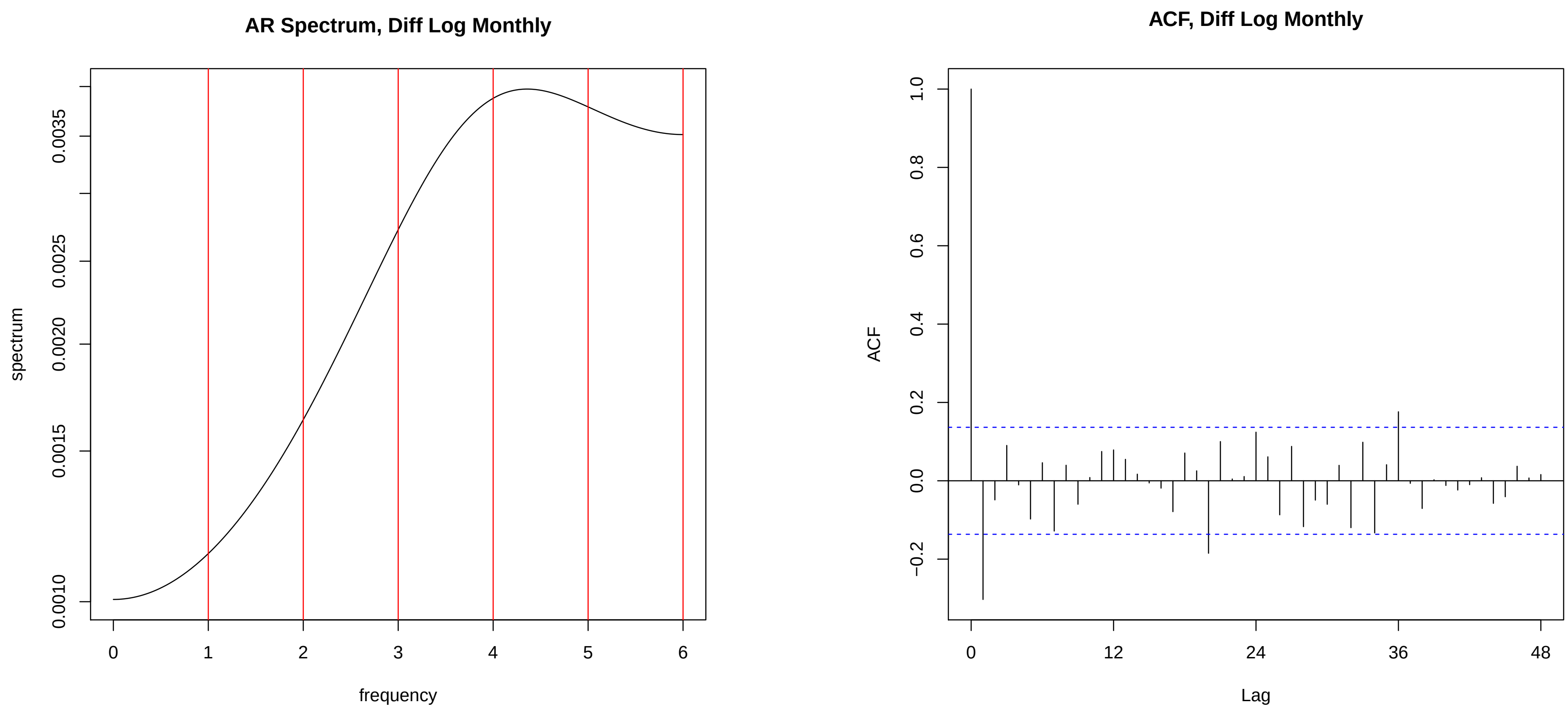

Figure 10: Autoregressive spectrum and autocorrelation function of the differenced log monthly series.

Example 2: Quarterly Aggregate

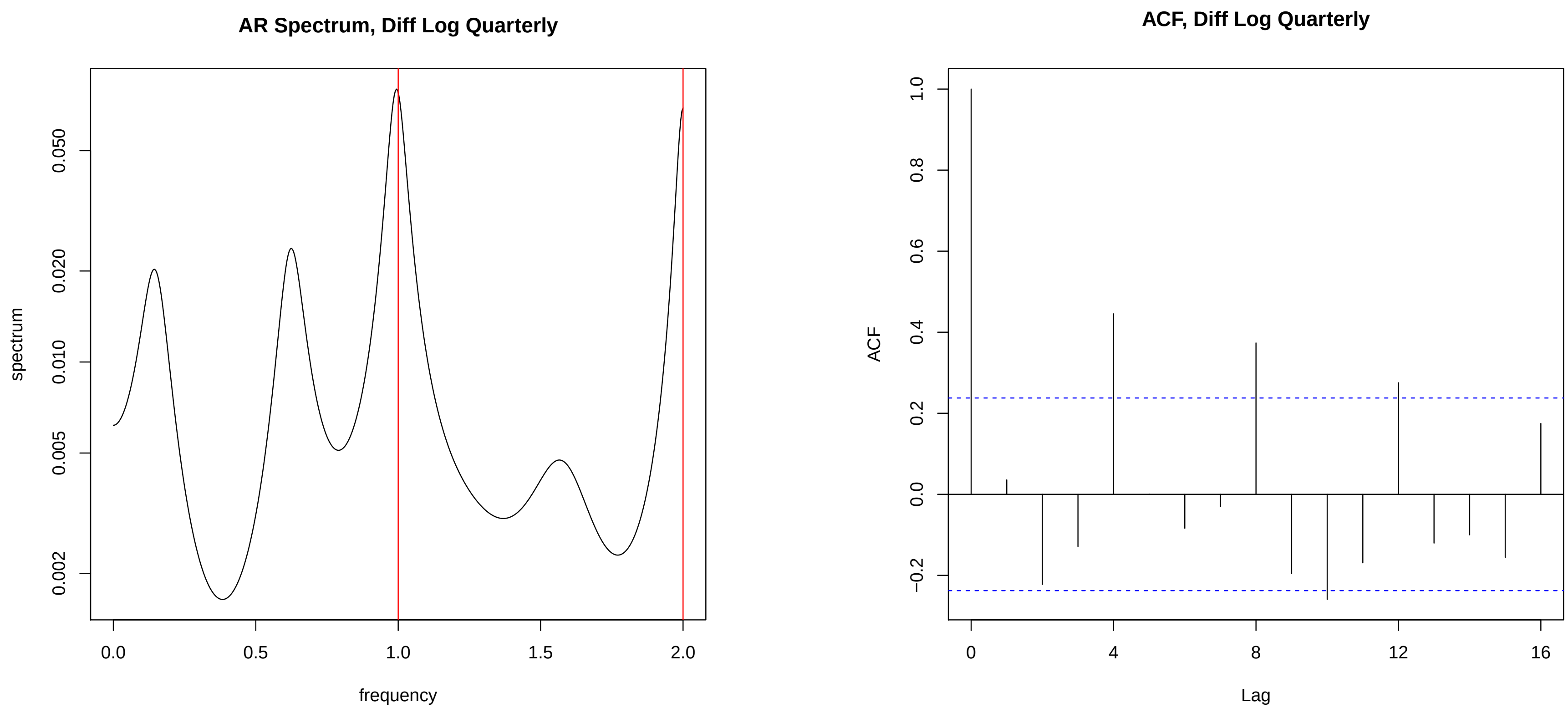

Figure 11: Autoregressive spectrum and autocorrelation function of the differenced log quarterly aggregate.

Example 2: Comments

- AR spectrum for monthly does not suggest seasonality (i.e., no peaks aligned with monthly seasonal frequencies); AR spectrum for quarterly has a sharp peak that appears to be located close to, if not on, quarterly seasonal frequency (red lines)

- ACF for monthly series may have significant autocorrelation at 3rd seasonal lag, but not either of the first two; ACF for quarterly series appears to show significant autocorrelations at each of first three seasonal lags

- That is, monthly series may not be seasonal, but aggregating suggests seasonality is noticeable at a quarterly frequency

- Table 1 shows values of ρ for which the specified series is deemed seasonal using the root diagnostic (i.e., the series exhibits ρ-persistent seasonality); again, quarterly aggregate and indirect quarterly seasonal adjustment values are similar

Series	ρ
Monthly	$\emptyset$
Qtrly Agg	$[0.980, 0.991]$
Monthly SA	$\emptyset$
Indirect Qtrly SA	$[0.980, 0.993]$
Direct Qtrly SA	$\emptyset$
Reconciled Mthly	$\emptyset$
Reconciled Qtrly	$\emptyset$

Table 2: Values of ρ for which the root diagnostic applied to the given series has a p-value exceeding $\alpha = 0.1$.

Example 2: Reconciled Monthly

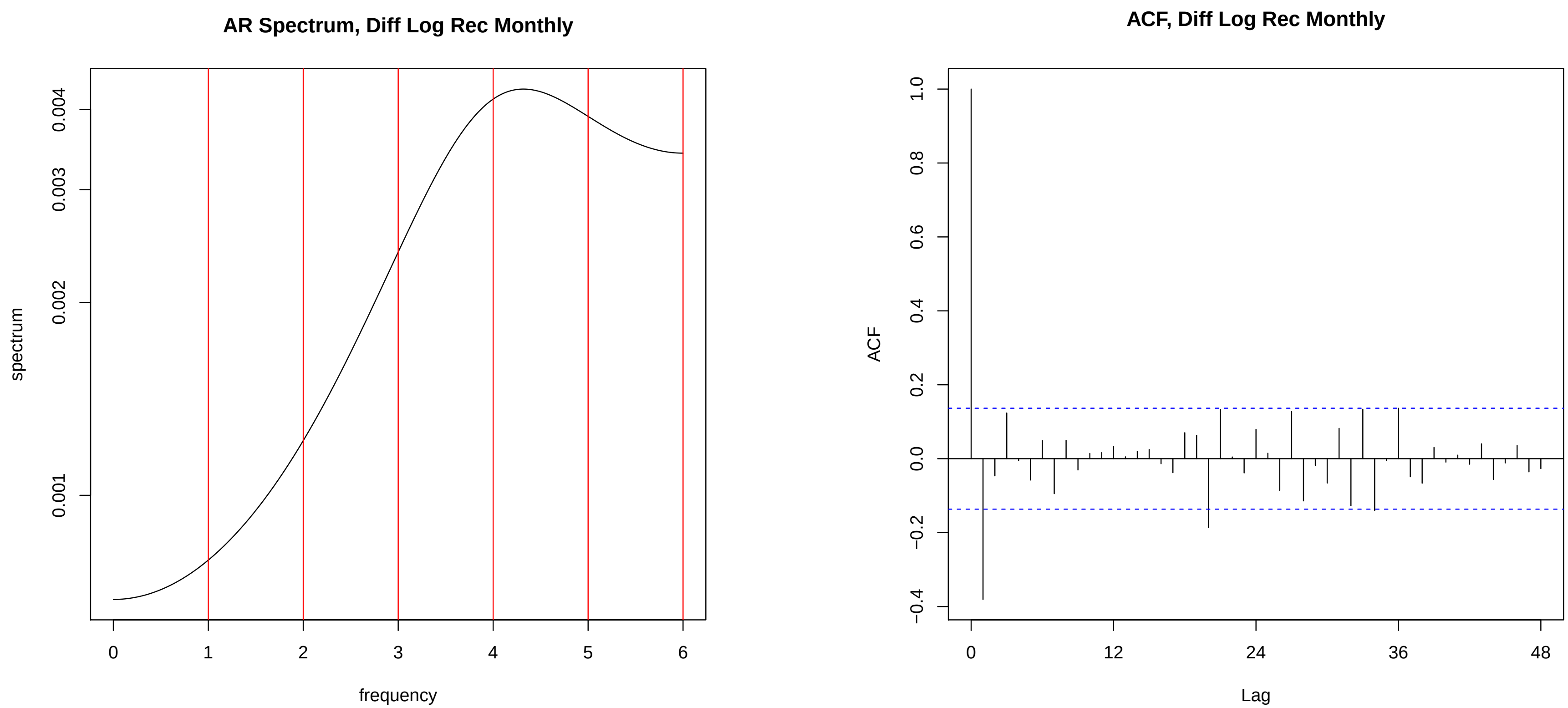

Figure 12: Autoregressive spectrum and autocorrelation function of the differenced log reconciled monthly series.

Example 2: Reconciled Quarterly

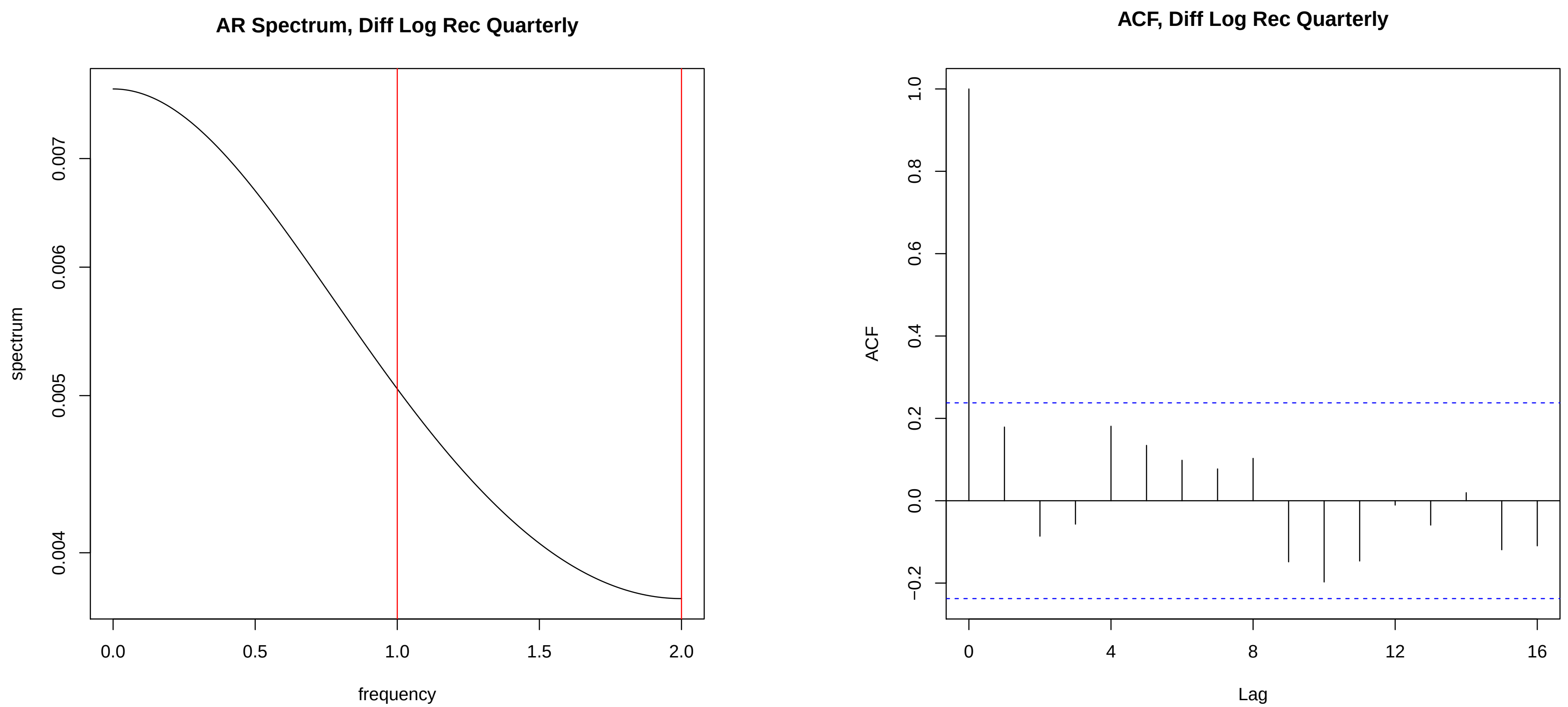

Figure 13: Autoregressive spectrum and autocorrelation function of the differenced log reconciled quarterly series.

Example 2: Monthly and Reconciled Monthly

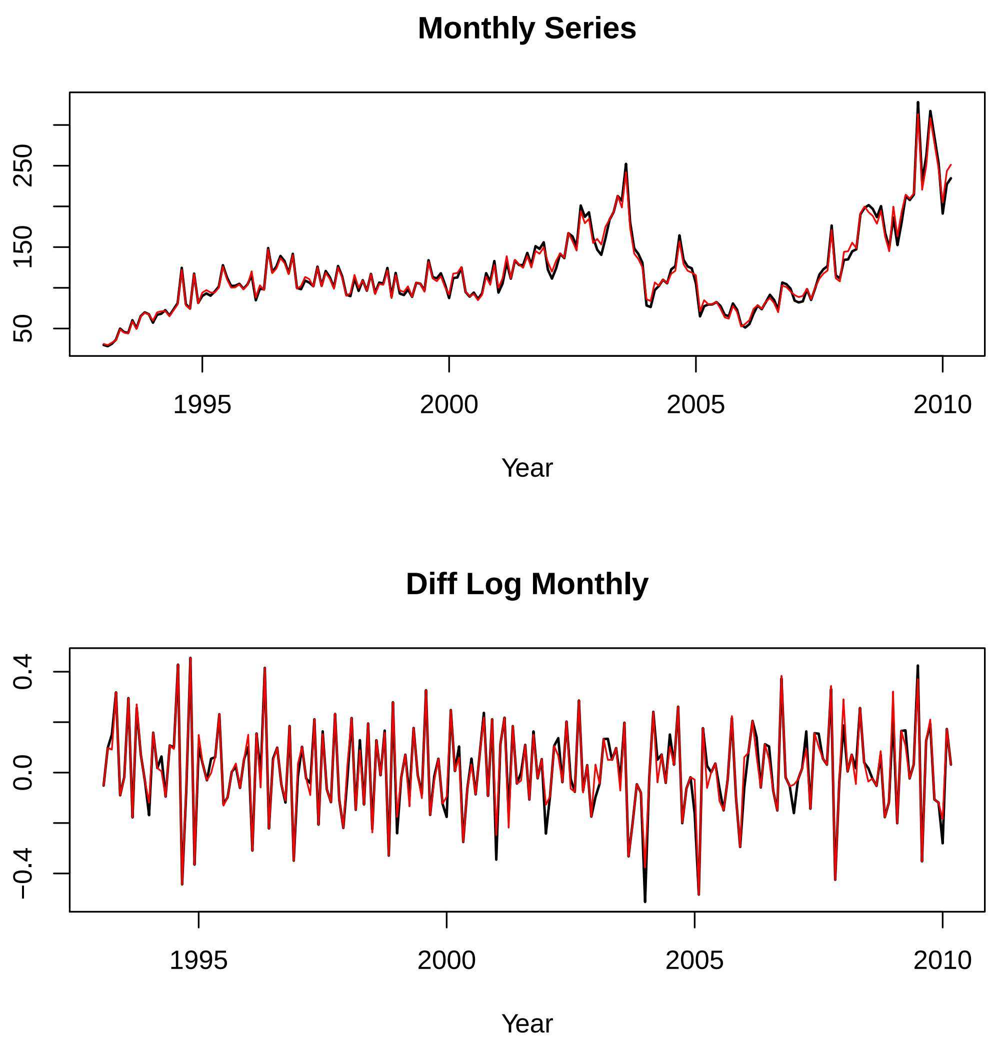

Figure 14: Monthly (black) and reconciled (red) series.

Example 2: Quarterly Aggregate and Reconciled Quarterly

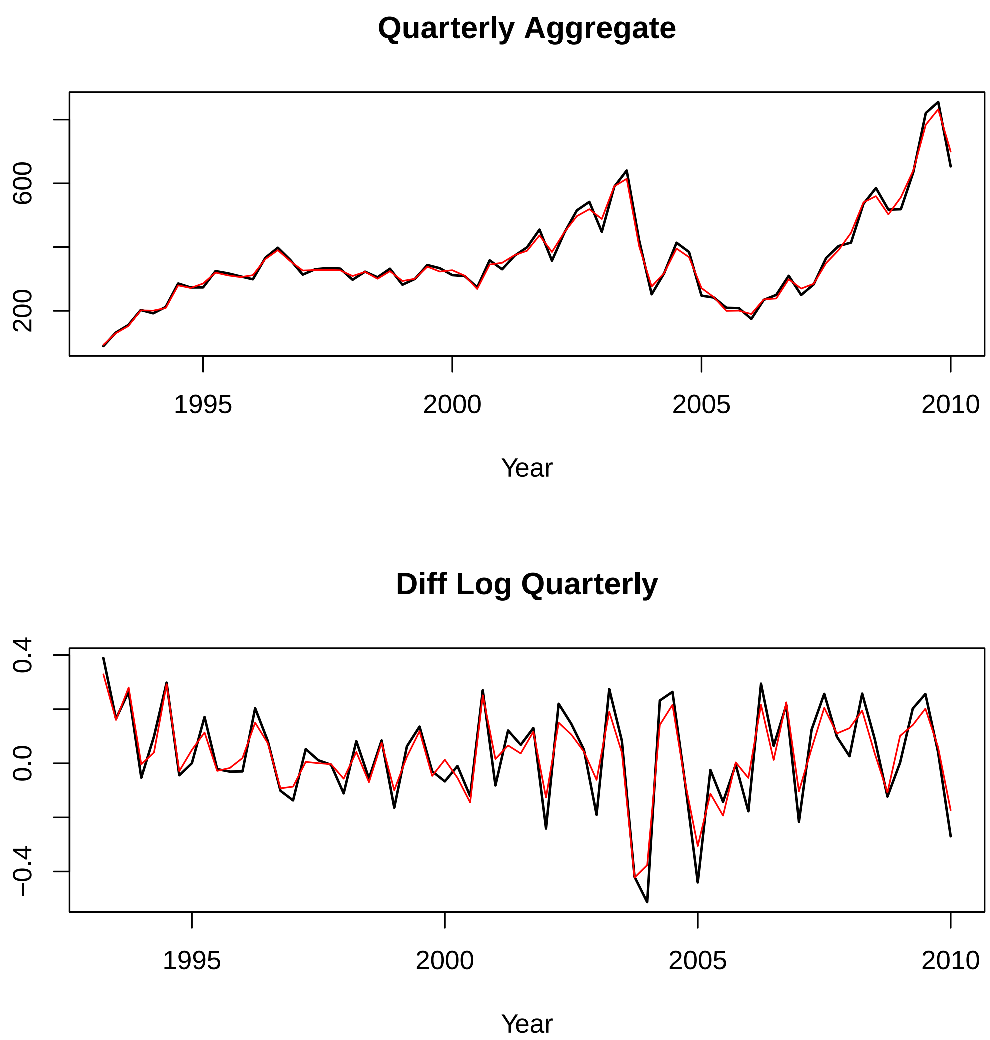

Figure 15: Quarterly aggregated (black) and reconciled (red) series.

Example 2: ... and Direct Quarterly Adjustment

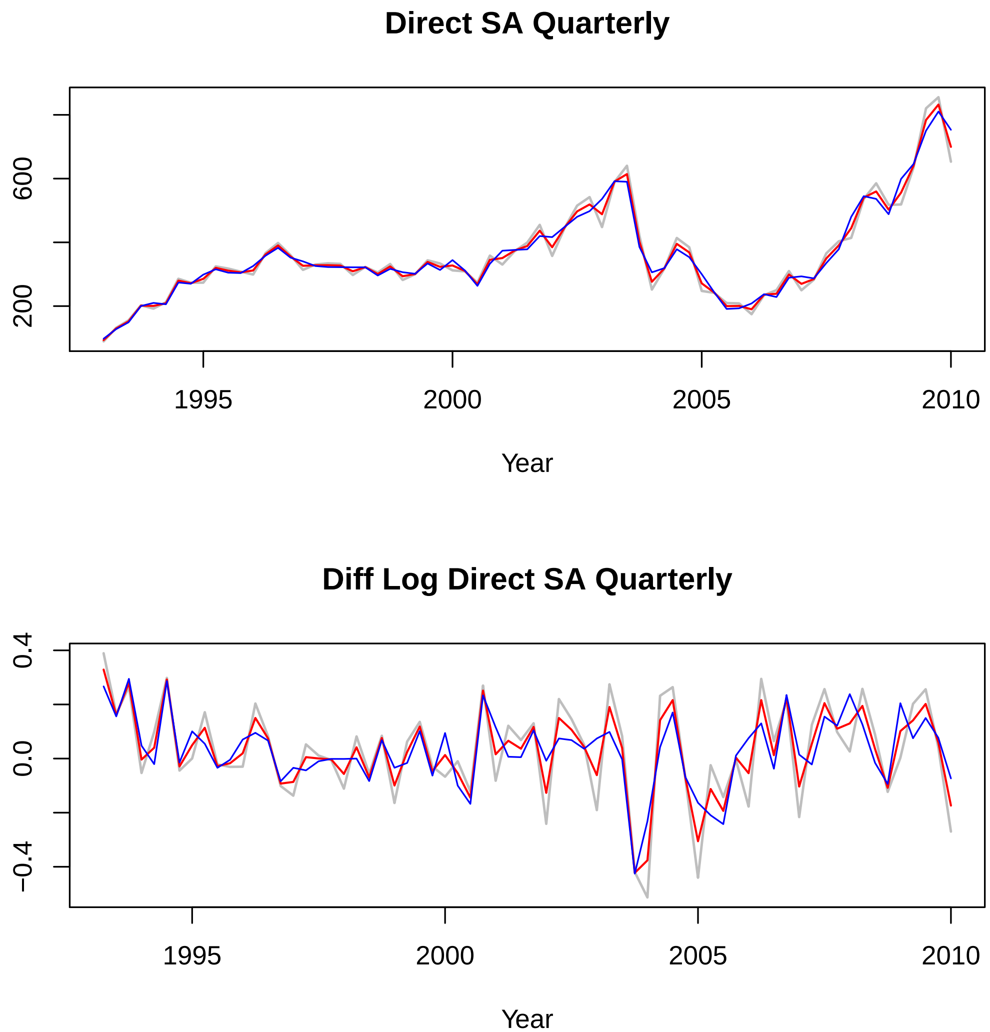

Figure 16: Quarterly aggregated (gray), reconciled (red), and directly adjusted quarterly (blue) series.

Thoughts, Future Steps

- Small changes observed to monthly series as a result of this process, but spectra and ACFs look the same (more or less)
- More noticeable changes to quarterly – reconciliation dampens some of the sharper changes in the aggregates
- Proviso: root diagnostic allows for choice of order for ARMA polynomial; examples used fixed values, but results may vary if order is chosen using some selection criterion (e.g., AIC) – currently in evaluation
- Optimization step is slowest – even if procedure finds adequate solutions at initial value set for ω_m and ω_q, time elapsed can be anywhere from 1 – 3 hrs; if possible, speeding this up would be useful for practical purposes
- Examples thus far have been with series where raw monthly is not adjusted and quarterly is noticeably seasonal; checking for series with different behavior

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