

# Tracking Cross-State Achievement Gap Trajectory in NAEP Assessments with Dynamic Time Warping Methods

**Qiwei (Britt) He**

Provost' Distinguished Associate Professor  
Data Science and Analytics Program  
AI Measurement and Data Science Lab  
Georgetown University



GEORGETOWN UNIVERSITY

# Content

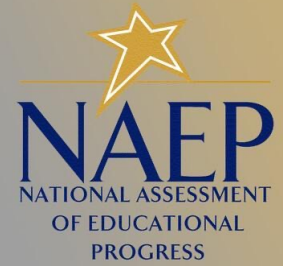
- **Overview**
  - National Report Card
  - Cross-State Achievement Gap Trajectories in NAEP
- **Methods**
  - Dynamic Time Warping Similarity Measurement
  - Sequence Clustering
- **Results**
  - Trajectory patterns across states
  - Achievement gap changes across grades and subjects
- **Discussion**

# 1

## Overview

- National Report Card
- Cross-State Achievement Gap Trajectories

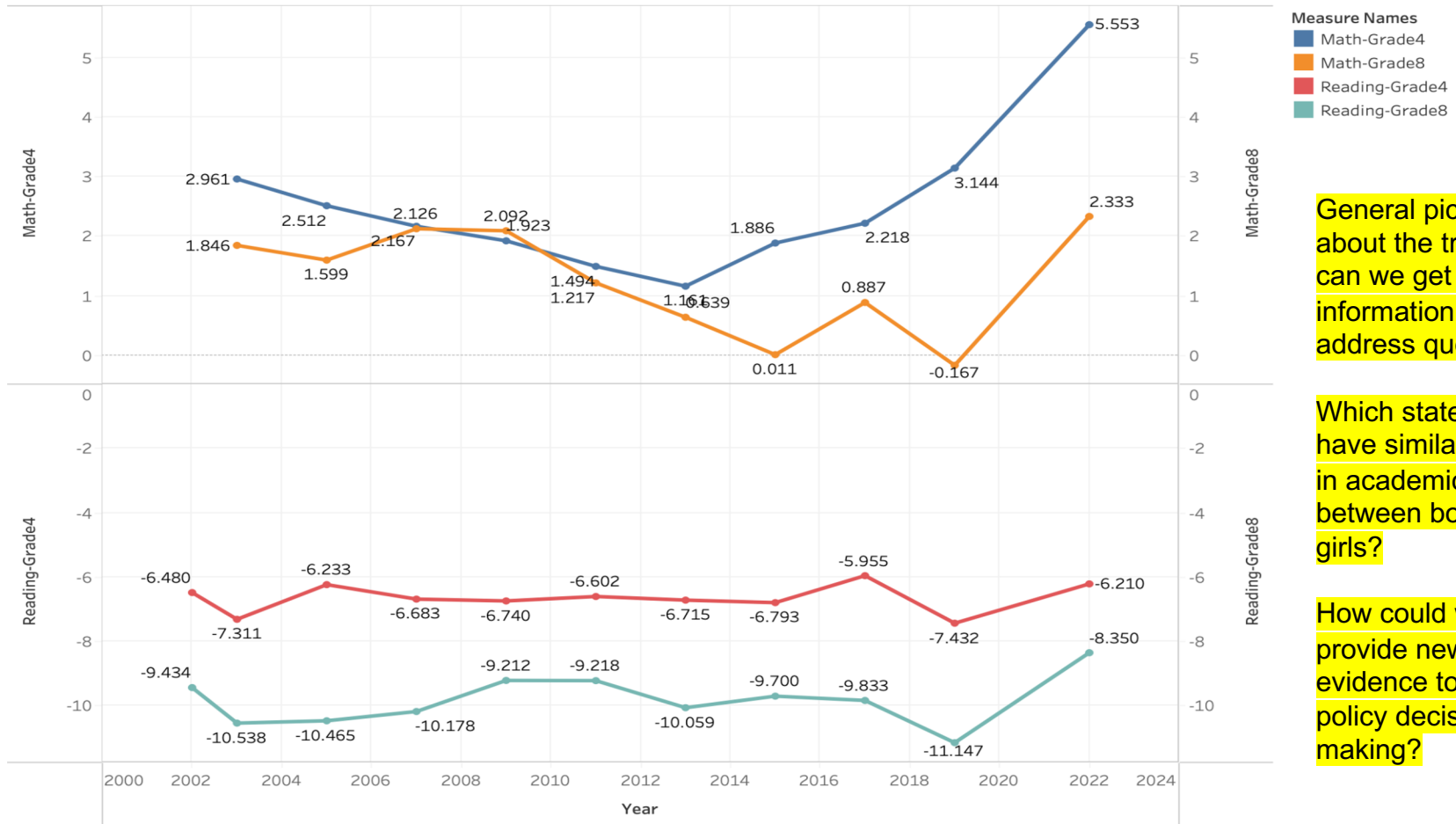
# National Report Card



- Educational assessment plays **a crucial role** in uncovering learning disparities and shaping data-informed education policy (NCES, 2023).
- The **National Assessment of Educational Progress (NAEP)** is a nationally representative and continuing assessment of what America's students know and can do in various subjects, generally in Grade 4, 8, 12.
- Academic performance by different demographic groups, such as achievement gap between boys and girls, has aroused great attention since the start of NAEP.
- Within the past two decades, **over 42,900 journal papers** were published on the topic with "NAEP and demographic groups".
- However, **very few research** showed a **comprehensive picture through the NAEP development history and identify trajectory changes across states.**

# Overview of Academic Gap by Years and Subjects

## National Reading VS. Math Gender Gap



The trends of Math-Grade4, Math-Grade8, Reading-Grade4 and Reading-Grade8 for Year. Color shows details about Math-Grade4, Math-Grade8, Reading-Grade4 and Reading-Grade8.

General picture about the trajectory, can we get granular information to address questions:

Which state(s) may have similar patterns in academic gap between boys and girls?

How could we provide new evidence to support policy decision making?

# Objectives

- Track cross-state achievement gap trajectories in NAEP Assessments
- Identify trajectory patterns by boys and girls across grades and subjects
- Provide new evidence to policy makers through data-driven approach



# 2

## Methods

- Dynamic Time Warping Similarity Measurement
- Sequence Clustering

# Data

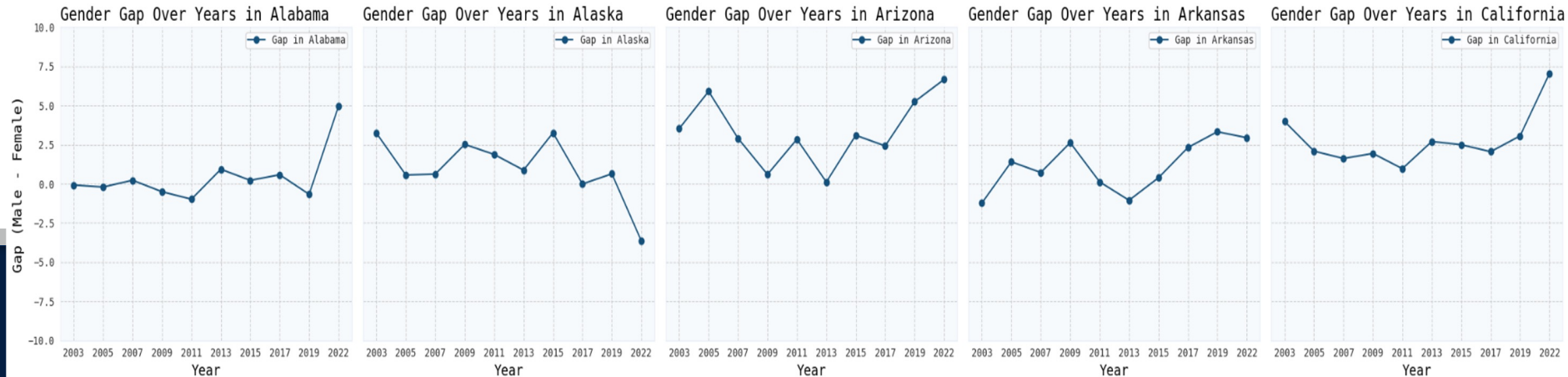
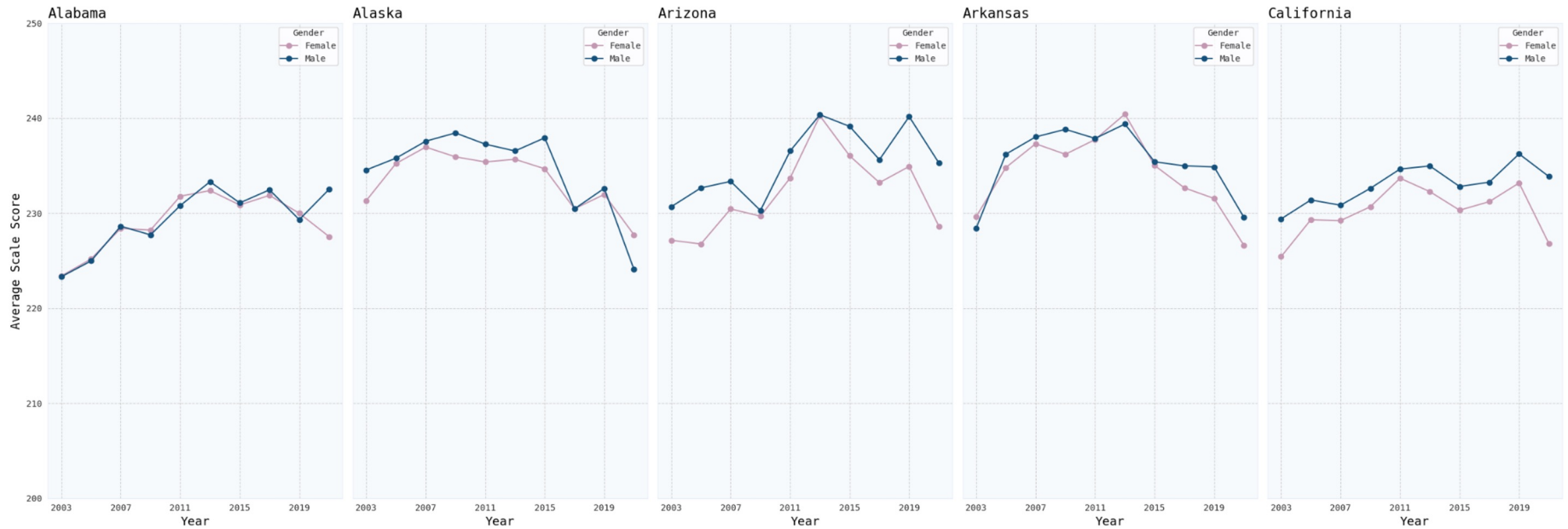
- **Subjects:** Math and Reading across 50 states
- **Grades:** Grade 4 and Grade 8
- **Years Covered:** 2002–2022
- **Gender gap:**  $M_{male} - M_{female}$  per cycle per subject
- Sequence data analysis





# Sequence Data by State (Grade 4 Math)

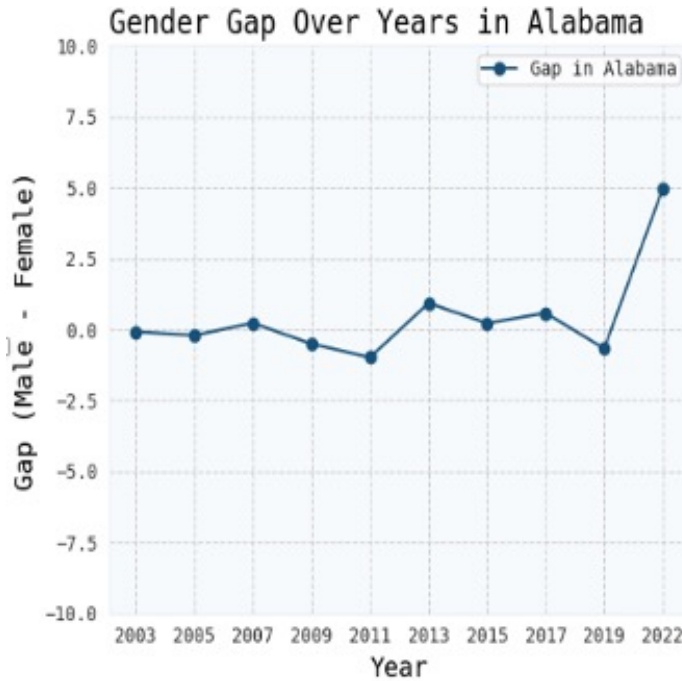
Average Mathematics Score By Gender Over Time for Selected States



# Sequence Data by State

## Gender difference across years in Grade 4 Math

|         | 2003  | 2005  | 2007 | 2009  | 2011  | 2013 | 2015 | 2017  | 2019  | 2022  |
|---------|-------|-------|------|-------|-------|------|------|-------|-------|-------|
| Alabama | -0.08 | -0.21 | 0.23 | -0.51 | -0.98 | 0.93 | 0.22 | 0.58  | -0.67 | 4.96  |
| Alaska  | 3.23  | 0.56  | 0.62 | 2.52  | 1.87  | 0.87 | 3.26 | -0.01 | 0.64  | -3.64 |



# Dynamic Time Warping (DTW)

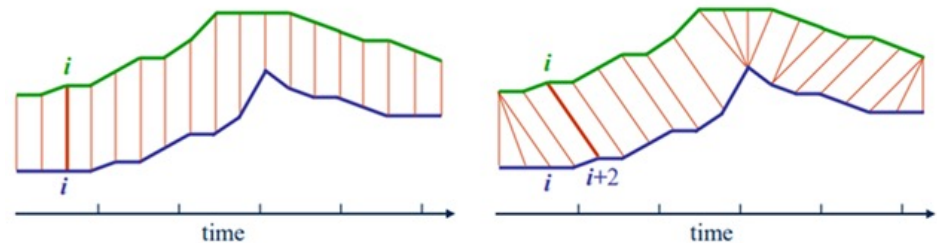
(He et al., 2023)

- Dynamic time warping (Sakoe & Chiba, 1978) is a distance measure that searches the optimal warping path between two series.

- Given sequences

$X = \{x_1, x_2, \dots, x_n\}$  and

$Y = \{y_1, y_2, \dots, y_m\}$  with the same or different lengths, a warping path  $W$  is an alignment between  $X$  and  $Y$ , involving one-to-many mappings for each pair of elements.



Any distance (Euclidean, Manhattan, ...) which aligns the  $i$ -th point on one time series with the  $i$ -th point on the other will produce a **poor similarity score**.

A non-linear (elastic) alignment produces a **more intuitive similarity measure**, allowing similar shapes to match even if they are out of phase in the time axis.

# Dynamic Time Warping Algorithm

- The initial step of DTW algorithm is defined as:

$$DTW(i, j) = \begin{cases} \infty & \text{if } (i = 0 \text{ or } j = 0) \text{ and } i \neq j \\ 0 & \text{if } i = j = 0 \end{cases}$$

- The recursive function of DTW is defined as

$$DTW(i, j) = \min \begin{cases} DTW(i-1, j) + w_h C(i, j) \\ DTW(i, j-1) + w_v C(i, j) \\ DTW(i-1, j-1) + w_d C(i, j) \end{cases}$$

where  $(w_h, w_v, w_d)$  are weights for the horizontal, vertical and diagonal directions, respectively.  $DTW(i, j)$  denotes the distance or cost between two sub-sequences  $\{x_1, x_2, \dots, x_i\}$  and  $\{y_1, y_2, \dots, y_j\}$ , and  $DTW(N, M)$  indicates the total cost of the optimal warping path.

|   |   |    |   |   |   |   |   |    |
|---|---|----|---|---|---|---|---|----|
| A | 5 | 10 | 6 | 3 | 1 | 2 | 4 | 3  |
|   | 4 | 6  | 3 | 1 | 0 | 1 | 3 | 3  |
|   | 3 | 3  | 1 | 0 | 1 | 1 | 2 | 4  |
|   | 2 | 1  | 0 | 1 | 3 | 4 | 4 | 7  |
|   | 1 | 0  | 1 | 3 | 6 | 8 | 9 | 13 |
|   |   | 1  | 2 | 3 | 4 | 3 | 2 | 5  |
|   |   | B  |   |   |   |   |   |    |

$A_i = \{1, 2, 3, 4, 5\}$

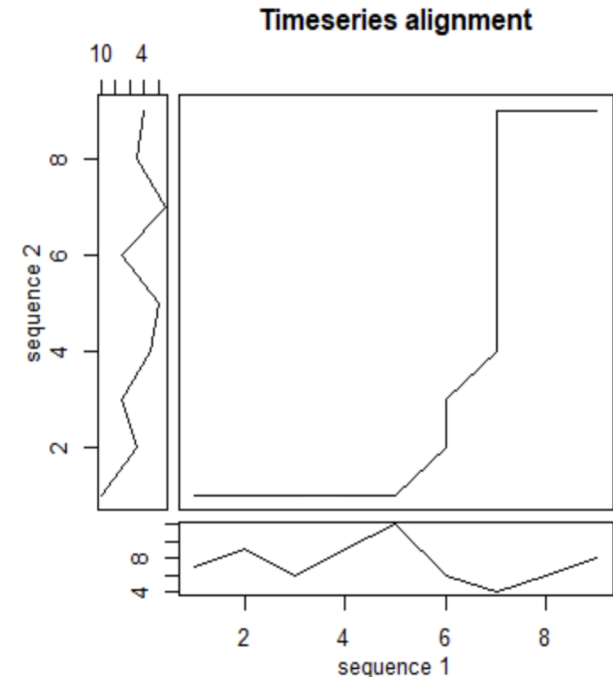
$B_j = \{1, 2, 3, 4, 3, 2, 5\}$

$$\begin{aligned} dtw(i, j) &= |A_i - B_j| + \min(D[i-1, j-1], D[i-1, j], D[i, j-1]) \\ &= |5 - 2| + 3 \end{aligned}$$

# Dynamic Time Warping Similarity

## Dynamic Time Warping (DTW)

- A time-series alignment algorithm to measure similarity between sequences that may vary in time or speed.
- Useful when states drop from certain cycles, causing unequal sequence lengths.
- DTW constructs a cost matrix and finds the optimal warping path between two sequences.
- Only applicable to numeric sequences, so categorical data needs to be numerically encoded.



# Sequence Clustering

## Sequence Clustering

- Performed on the DTW distance matrix.
- Uses partition-based clustering (K-medoids), where clusters are formed based on minimizing within-cluster DTW distances.
  - Starts by randomly selecting representative points (medoids).
  - Assigns other sequences to the nearest medoid.
  - Repeats until cluster centers stabilize.
- **Cluster Validity Indices Used**
  - **Silhouette Index:** Measures how similar an object is to its cluster vs. others.
  - **Dunn Index:** High inter-cluster, low intra-cluster distances preferred.
  - **Davies-Bouldin Index:** Measures cluster compactness and separation.
- Iterated values of  $K = 2$  to 10, selected **4 clusters** as optimal

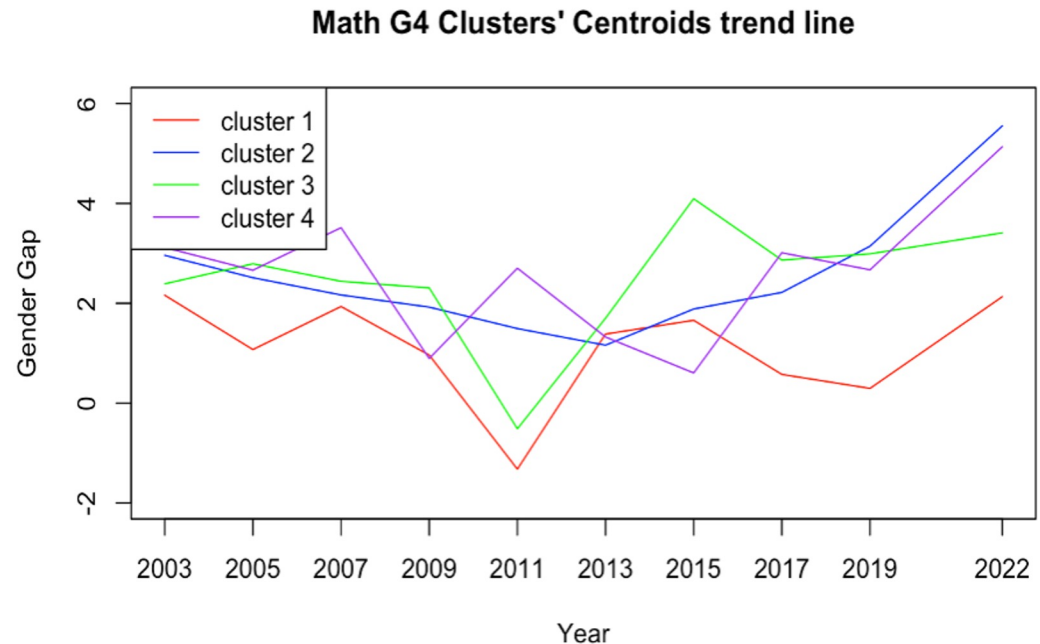
# 3

## Results

- Trajectory patterns across states
- Achievement gap changes across grades and subjects

# Trajectory Pattern Across States – Grade 4 Math

- **Cluster 1 (Red):**  
Significant drop in 2011 and 2015–2019, sharp increase post-2019.
- **Cluster 2 (Blue):**  
Relatively stable from 2003–2017, but steeply increase after 2017.
- **Cluster 3 (Green):** Similar shape as Cluster 1 but with not significantly impact by COVID.
- **Cluster 4 (Purple):** Up and down in big waves but steeply rises after 2019.





# Trajectory Pattern Across States – Grade 4 Math

## Cluster 1 : (8 States)

Alabama, Alaska, Georgia, Hawaii, Louisiana, Mississippi, North Carolina, West Virginia

## Cluster 2 : (21 States)

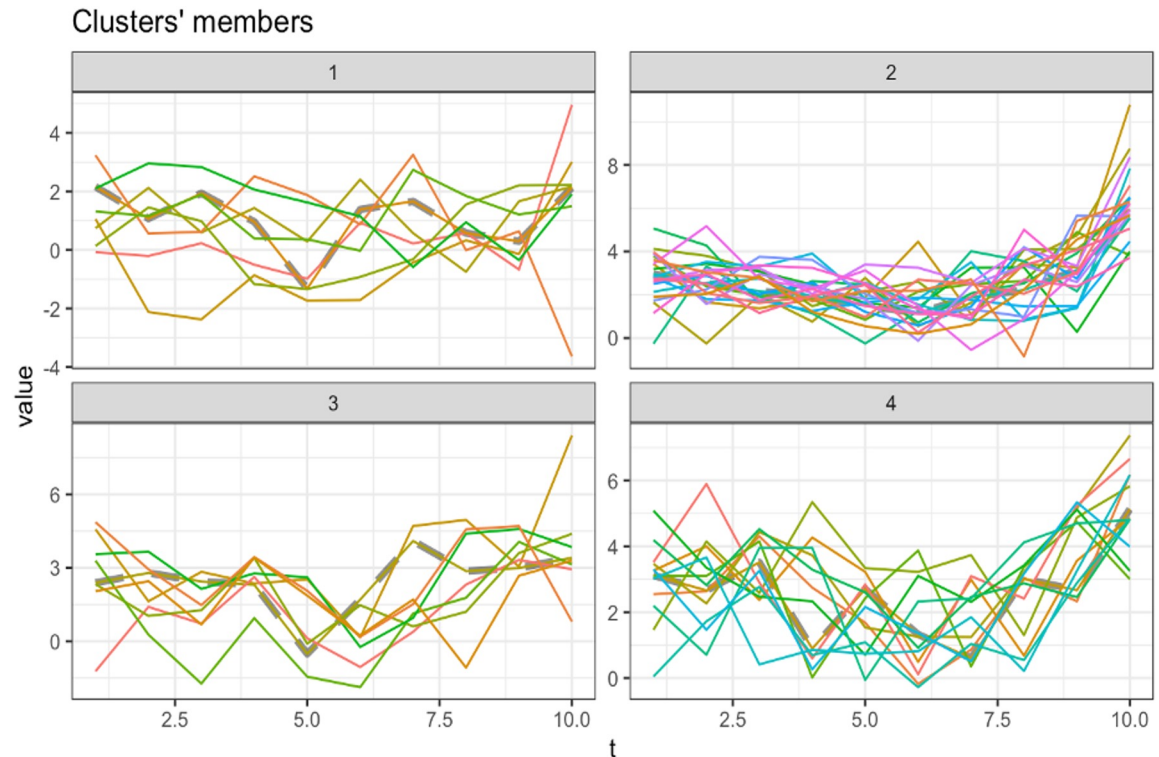
California, Colorado, Florida, Idaho, Indiana, Iowa, Kansas, Kentucky, Michigan, Missouri, Nevada, New Jersey, New Mexico, New York, Ohio, Pennsylvania, Utah, Vermont, Virginia, Wisconsin, Wyoming

## Cluster 3 : (8 States)

Arkansas, Connecticut, Delaware, Massachusetts, Oklahoma, Rhode Island, South Carolina, South Dakota"

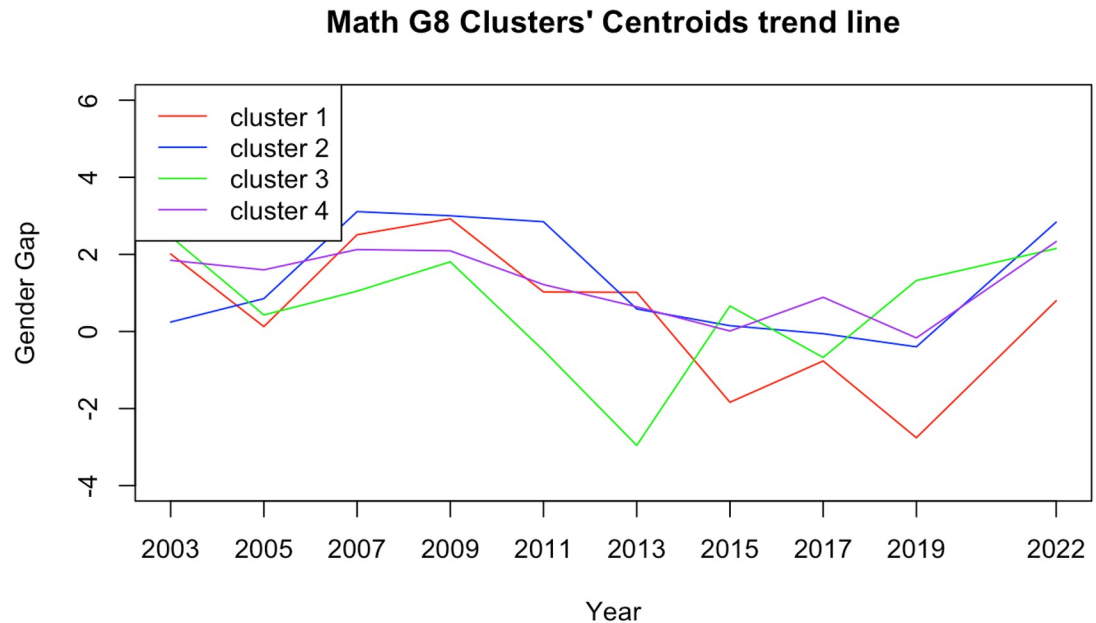
## Cluster 4 : (13 States)

Arizona, Illinois, Maine, Maryland, Minnesota, Montana, Nebraska, New Hampshire, North Dakota, Oregon, Tennessee, Texas, Washington



# Trajectory Pattern Across States – Grade 8 Math

- **Cluster 1 (Red):** significantly dropped between 2015 and 2019, but sharply increased after 2019.
- **Cluster 2 (Blue):** Stable and a bit down between 2003 and 2017, sharply increased after 2019.
- **Cluster 3 (Green):** Gap dropped to the lowest value in 2013, and kept fast increasing afterwards.
- **Cluster 4 (Purple):** Stable and a small dropping trend until 2019, where a noticeable spike appeared after 2019.



# Trajectory Pattern Across States – Grade 8 Math

## Cluster 1 : (10 States)

Kentucky, Maryland, Massachusetts, North Carolina, Ohio, Oregon, Pennsylvania, South Carolina, South Dakota, Wyoming

## Cluster 2 : (14 States)

Arizona, Arkansas, Colorado, Kansas, Maine, Michigan, Montana, Nebraska, North Dakota, Oklahoma, Tennessee, Vermont, West Virginia, Wisconsin

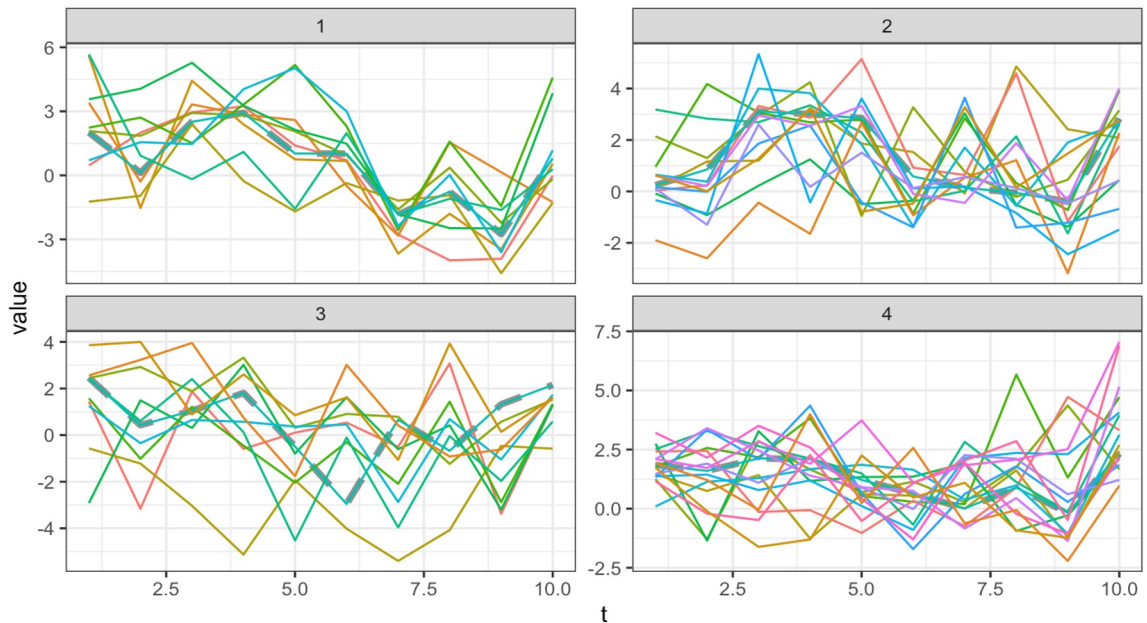
## Cluster 3 : (10 States)

Alabama, Delaware, Florida, Hawaii, Indiana, Louisiana, Minnesota, Mississippi, New York, Rhode Island

## Cluster 4 : (16 States)

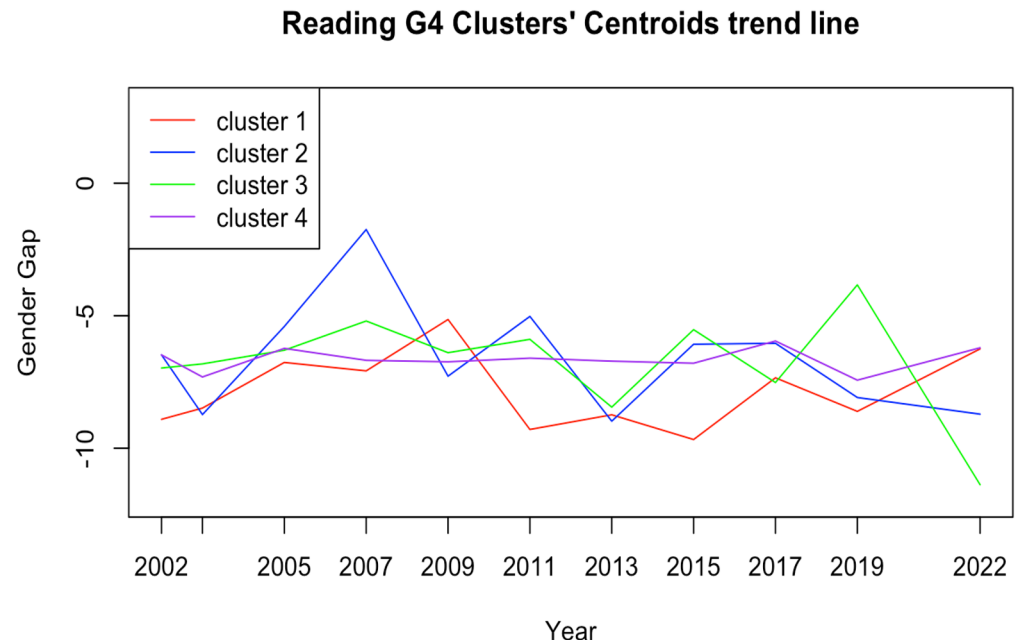
Alaska, California, Connecticut, Georgia, Idaho, Illinois, Iowa, Missouri, Nevada, New Hampshire, New Jersey, New Mexico, Texas, Utah, Virginia, Washington

Clusters' members



# Trajectory Pattern Across States – Grade 4 Reading

- **Cluster 1 (Red):** Consistently large gap (girls outperform over boys), gap slightly narrowed after 2019.
- **Cluster 2 (Blue):** High peak around 2007 indicating the smallest gender gap, curve waves showing in the recent decade.
- **Cluster 3 (Green):** Fairly stable until 2017, with a sharp spike in 2019 (with the smallest gender gap), followed by a sharp drop in 2022 after COVID.
- **Cluster 4 (Purple):** Consistently stable with moderate gender gap across all years.



# Trajectory Pattern Across States – Grade 4 Reading

## Cluster 1 : (11 States)

Alabama, Florida, Hawaii, Indiana, Missouri, Oregon, Pennsylvania, South Carolina, Vermont, Washington, West Virginia

## Cluster 2 : (4 States)

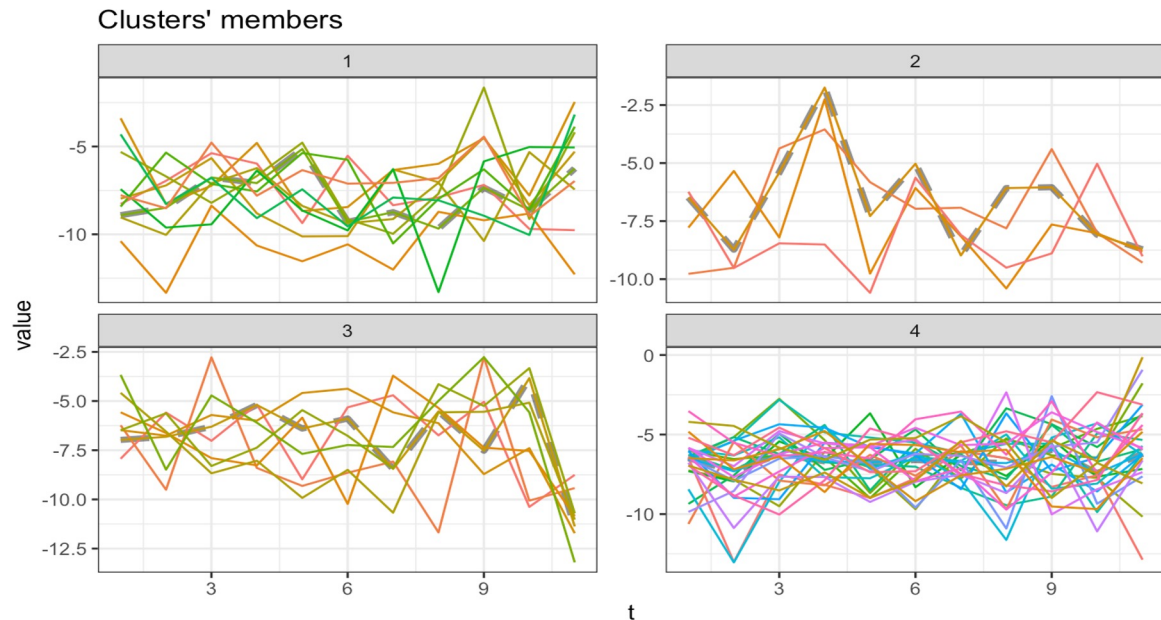
Arkansas, Montana, New Mexico, Wisconsin

## Cluster 3 : (8 States)

Idaho, Louisiana, Mississippi, New Jersey, Oklahoma, Rhode Island, South Dakota, Virginia

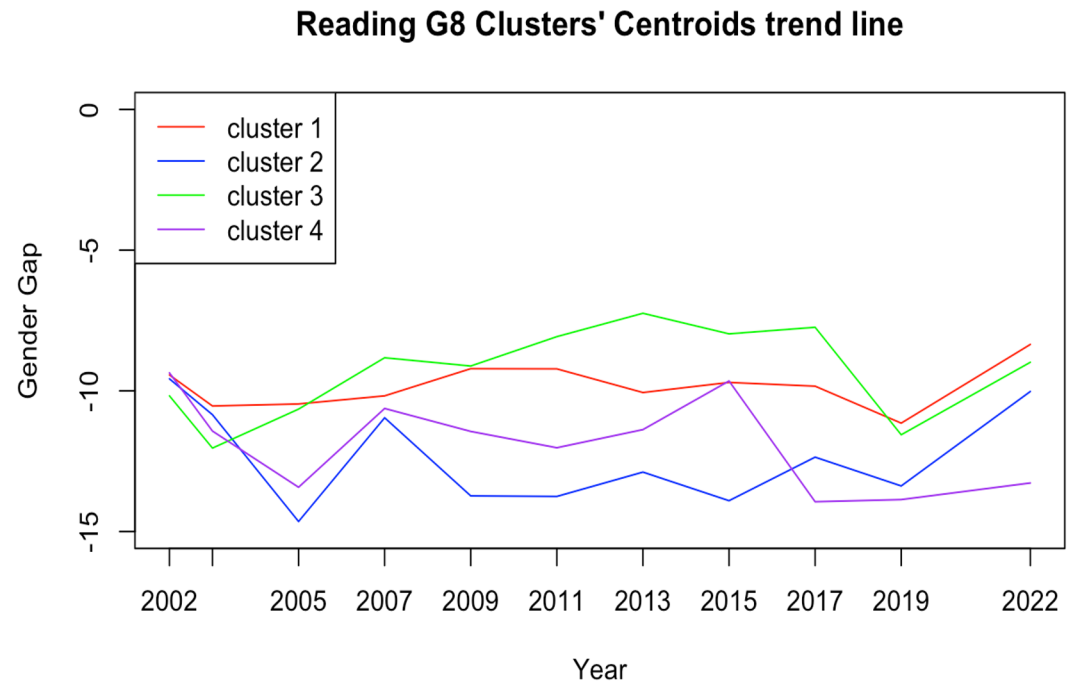
## Cluster 4 : (27 States)

Alaska, Arizona, California, Colorado, Connecticut, Delaware, Georgia, Illinois, Iowa, Kansas, Kentucky, Maine, Maryland, Massachusetts, Michigan, Minnesota, Nebraska, Nevada, New Hampshire, New York, North Carolina, North Dakota, Ohio, Tennessee, Texas, Utah, Wyoming



# Trajectory Pattern Across States – Grade 8 Reading

- **Cluster 1 (Red):** Consistently wide gender gap across years, with a slight narrowing in 2013–2017 and a small uptick after 2019.
- **Cluster 2 (Blue):** The largest gender gap, esp. the lowest point at 2005, the gap slightly narrowed down after 2017.
- **Cluster 3 (Green):** The narrowest gender gap among clusters between 2007 and 2015, a big drop in 2019, with a slight increasing trend (gap narrowing) in 2022.
- **Cluster 4 (Purple):** Moderate fluctuations with a noticeable drop in 2017, relatively stable afterwards.





# Trajectory Pattern Across States – Grade 8 Reading

## Cluster 1 : (20 States)

Arizona, Arkansas, California, Delaware, Florida, Georgia, Illinois, Indiana, Kentucky, Louisiana, Maryland, Michigan, Mississippi, New Jersey, Ohio, Oregon, Rhode Island, South Carolina, Utah, Virginia

## Cluster 2 : (5 States)

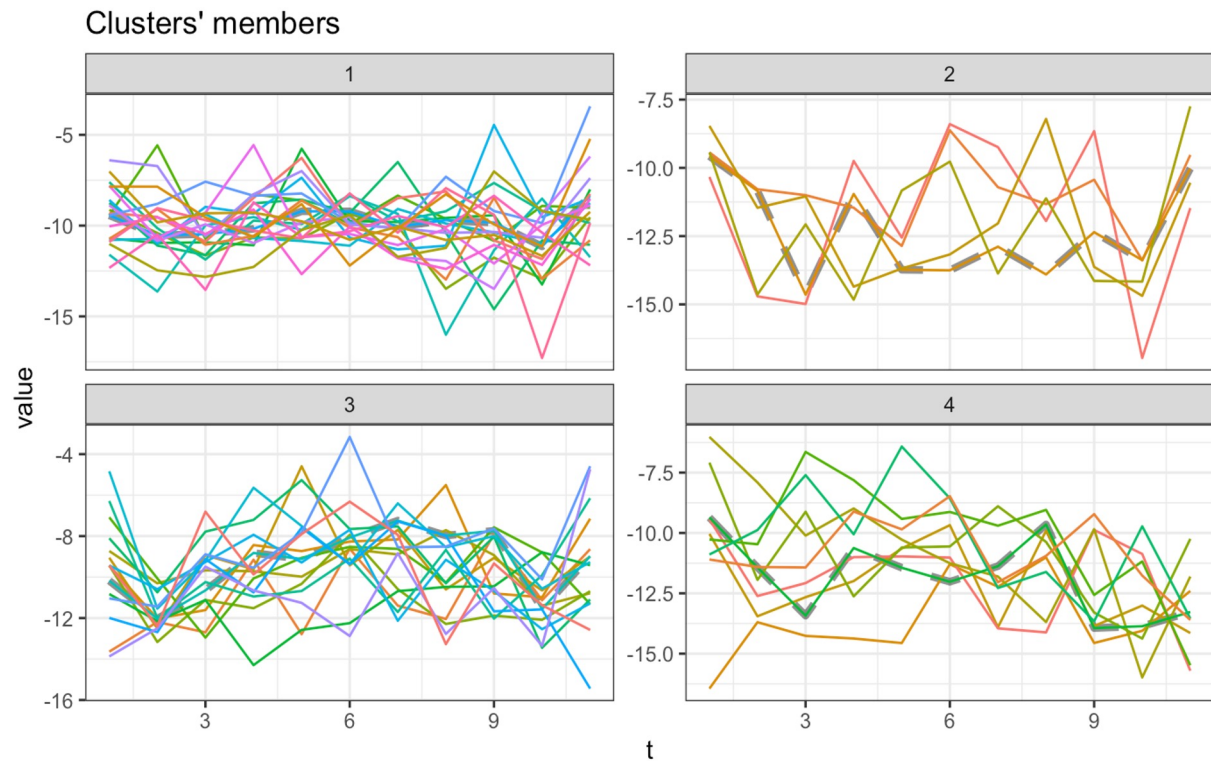
Alabama, New Hampshire, North Carolina, West Virginia, Wisconsin

## Cluster 3 : (16 States)

Colorado, Idaho, Iowa, Kansas, Massachusetts, Minnesota, Nebraska, Nevada, New Mexico, New York, Oklahoma, Pennsylvania, South Dakota, Tennessee, Texas, Washington

## Cluster 4 : (9 States)

Alaska, Connecticut, Hawaii, Maine, Missouri, Montana, North Dakota, Vermont, Wyoming



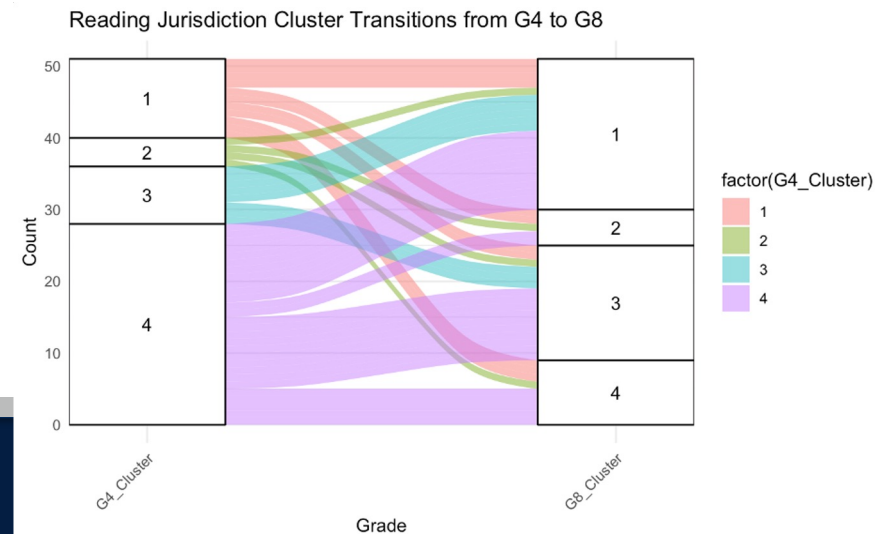
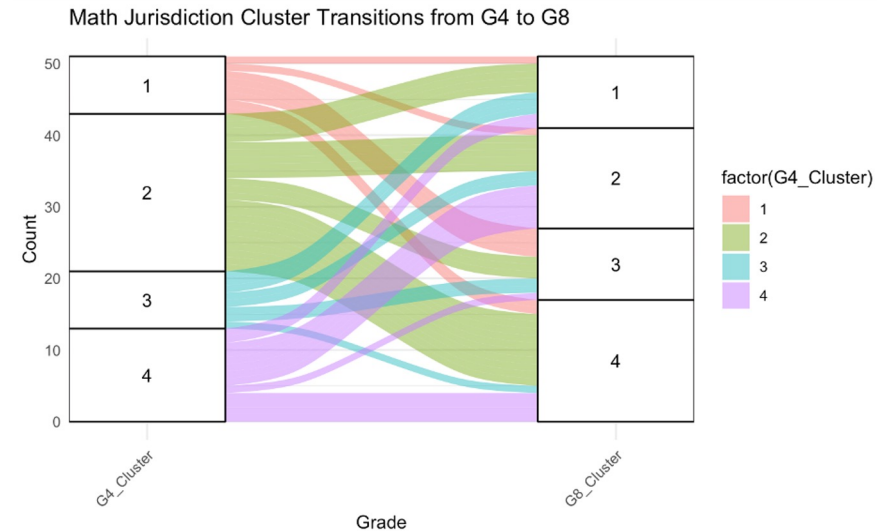
# Achievement Gap Changes by Grades and Subjects

**Alluvial Plots** used to visualize cluster shifts:

- Transition from Grade 4 to Grade 8
- Math vs Reading.

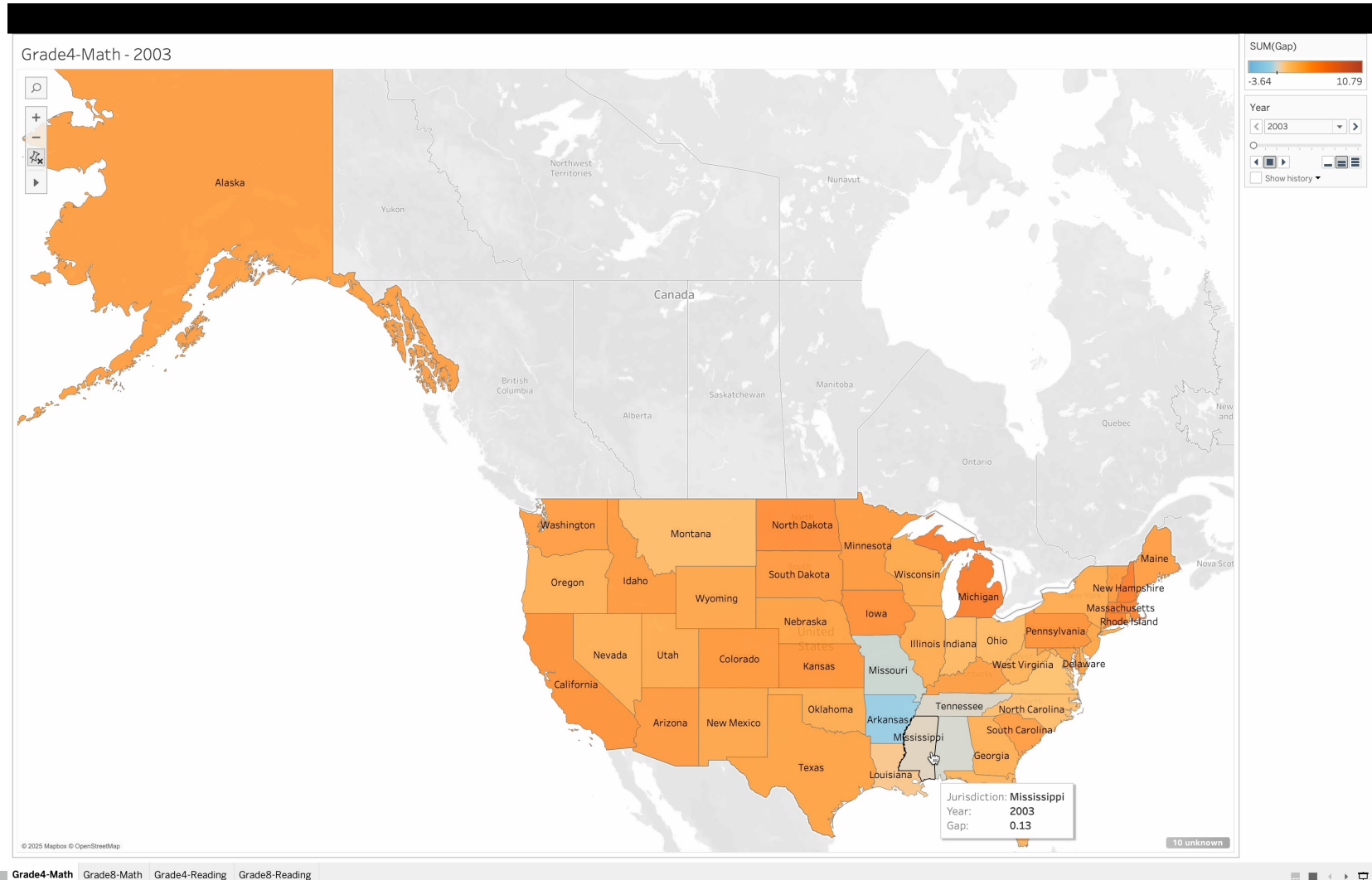
## Findings

- **Math** shows many cross-cluster transitions, indicating less stability across grade.
- **Reading** shows more jurisdictions remaining in the same cluster or nearby clusters.
- **Reading is more stable than Math** across grade 4 and grade 8.

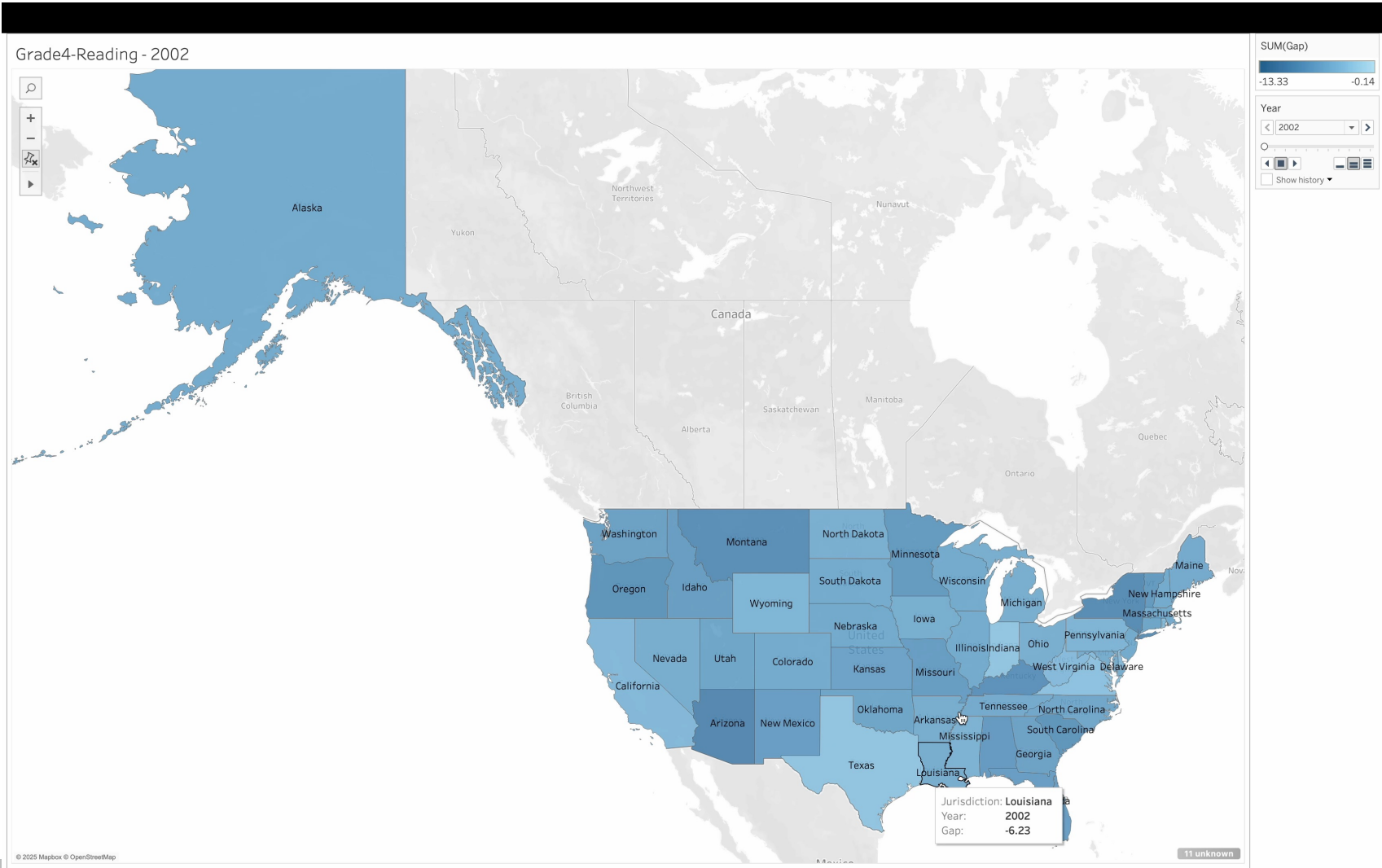




# Grade 4 Math Achievement Gap by Gender Across Years

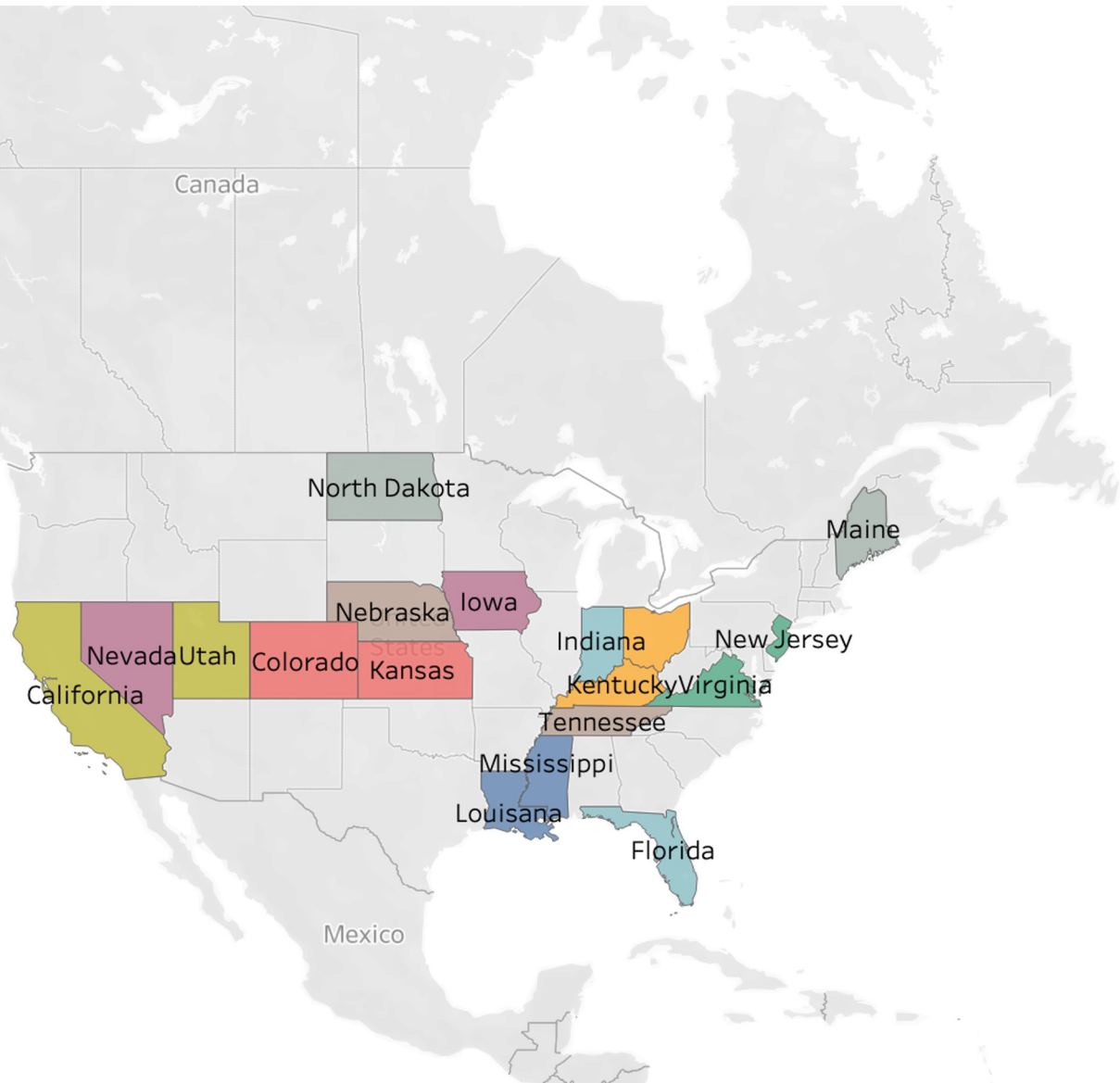


# Grade 4 Reading Achievement Gap by Gender Across Years



# Geographical Patterns shown in Achievement Gaps

- Nine state pairs consistently grouped together across all settings:
  - Kentucky & Ohio
  - California & Utah
  - Florida & Indiana
  - Louisiana & Mississippi
  - Colorado & Kansas
  - New Jersey & Virginia
  - Iowa & Nevada
  - Nebraska & Tennessee
  - North Dakota & Maine
- Indicates possible shared policies, demographics, or regional factors.



# 4 | Discussion

# Summary

- The sequence-based dynamic time warping similarity measurement and sequence clustering support **visualization on trajectory pattern** of achievement gaps across states, and identify **homogenous patterns across state clusters**.
- The achievement gaps between boys and girls significantly **enlarged in Math after COVID**.
- The achievement gaps between boys and girls is **less stable in math than in reading**, suggesting the educational policy or curriculum changes might impact more sensitively in math learning.
- The patterns of achievement gaps trajectory patterns are **more stable in Grade 8 than in Grade 4**, suggesting the younger group might be more impacted with the change of educational policy and curriculum development.

# Limitation and Future Work

- This study provides a novel perspective on the impact of educational policies on student performance. Future research could **incorporate additional demographic variables, such as school type, socio-economic status (SES), and region, to analyze trajectory patterns.**
- Methodology developed in this study can be **generally extended to school districts, cities,** to track trajectories of students' growth, academic gaps by different groups.
- Developing a **centralized portal or data platform** could serve as a **valuable tool for tracking performance trends** in NAEP and other assessments over time.
- Findings from this study should be linked **more closely to state-level educational policies** to identify and address the root causes of achievement gaps.





## AI Measurement and Data Science Lab

- Process data analytic methods
- AI item generation
- Psychiatry acuity measurement
- ML on missing data prediction
- Eye-tracking for ELL
- VR/AR in assessment

# Thank you very much!

Qiwei (Britt) He, PhD  
[qiwei.he@georgetown.edu](mailto:qiwei.he@georgetown.edu)



GEORGETOWN UNIVERSITY