An F Test for the Presence of Moving Seasonality When Using Census Method II-X-11 Variant

by
John Higginson
Seasonal Adjustment Methods
Statistics Canada
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An analysis-of-variance F test for the presence of moving seasonality characterized by gradual changes in the amplitude is performed on a modification of the seasonal-irregular ratios or differences obtained from Table D.8 of Census Method II-X-11 variant.

The statistical basis for this test is given in Scheffé, Henry, The Analysis of Variance, J. Wiley, 1959.

1. Monthly Moving Seasonality Test for Additive Models

Let $(S+I)_{ij}$ denote the seasonal irregular difference from Table D.8 corresponding to year i, i=1, 2, ..., N and month j, j=1, 2, ..., 12. The test replaces each SI difference by its absolute value and then performs a two-way, or randomized block, analysis of variance on the transformed data.

The theoretical model adopted for studying these differences is given by the following equation:

(1) $(S+I)_{ij}=a_i+b_i+e_{ij}$

Equation (1) signifies that the value $(S+I)_{ij}$ for each difference represents the sum of:

- a term a_i representing the numerical contribution due to the effect of the ith year
- a term b_j representing the numerical contribution due to the effect of the jth month
- a residual component e_{ij} known as the irregular component, assumed to be normally distributed with zero mean, constant variance and zero covariance. It represents the effect on the values of the seasonal irregular differences of the whole set of factors not explicitly taken into account in the model.

The variance analysis is based on the decomposition of the total variance of the observations as a sum of the partial variances σ_m^2, σ_y^2, and σ_r^2.

$\sigma_m^2 = \frac{N}{11}\Sigma_j\ (\overline{X}_{.j} - \overline{\overline{X}})^2$ is the "between months" variance of the transformed data. It is the column, or treatment, effect that measures the magnitude of the seasonality, where $\overline{X}_{.j} = \frac{1}{N}\Sigma_{i=1}^{n} \mid S + I\mid_{ij}$ and $\overline{\overline{X}}=\Sigma_{ij}\mid S+I\mid_{ij}/12N$.

$\sigma_y^2 = \frac{11}{N-1}\ \Sigma_i\ (\overline{X}_{i.} - \overline{\overline{X}})^2$ is the "between years" variance of the transformed data. It is the row or block effect that measures the movement of the seasonality, where $\overline{X}_{i.} = \frac{1}{12}\ \Sigma_j \mid S + I\mid_{ij}$.

$\sigma_r^2 = \frac{1}{11(N-1)}\ \Sigma ij\ (\mid S + I\mid_{ij} - \overline{X}_{.j} - \overline{X}_{i.} + \overline{\overline{X}})^2$ is the "residual" variance.

The null hypothesis $a_1 = a_2 = a_n$, i.e. the one in which the effect of the years in the seasonality does not change can be checked by the ratio $F = \sigma_y^2/\sigma_r^2$.

This ratio is related to the Fisher-Snedecor's test with 11 and 11 (N-1) degrees of freedom. The F value is compared with the one in the table for the F test at a 1% level of significance for a series of 10 years of observations (the differences in values for series of different lengths are small).

A table of the analysis of variance is printed at the end of Table D.8 and also a message that indicates whether moving seasonality is present or not.

2. Monthly Moving Seasonality Test for Multiplicative Models

The test is analogues to the previous one except that now the seasonal irregular ratios SI are replaced by $|SI - 100|$.

The moving seasonality tests for quarterly data are analoous to the monthly tests.

The presence of moving seasonality in a series may lead to poor forecasts and large revisions. In some cases, it can be avoided or significantly reduced by switching from the multiplicative to the additive model or vice versa.