An F Test for the Presence of Moving Seasonality
When Using Census Method II-X-11 Variant

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An analysis-of-variance F test for the presence of moving seasonality
characterized by gradual changes in the amplitude is performed on a
modification of the seasonal-irregular ratios or differences obtained
from Table D.8 of Census Method II-X-11 variant.
The statistical basis for this test is given in Scheffé, Henry,

1. Monthly Moving Seasonality Test for Additive Models

Let \((S+I)_{ij}\) denote the seasonal irregular difference from Table D.8
corresponding to year \(i\), \(i=1, 2, ..., N\) and month \(j\), \(j=1, 2, ..., 12\).
The test replaces each SI difference by its absolute value and then
performs a two-way, or randomized block, analysis of variance on the
transformed data.

The theoretical model adopted for studying these differences is given
by the following equation:

\[
(1) \quad (S+I)_{ij} = a_i + b_j + e_{ij}
\]

Equation (1) signifies that the value \((S+I)_{ij}\) for each difference
represents the sum of:
- a term \( a_i \) representing the numerical contribution due to the
effect of the \( i \)th year
- a term \( b_j \) representing the numerical contribution due to the
effect of the \( j \)th month
- a residual component \( e_{ij} \) known as the irregular component, assumed
to be normally distributed with zero mean, constant variance and
zero covariance. It represents the effect on the values of the seasonal
irregular differences of the whole set of factors not explicitly taken
into account in the model.

The variance analysis is based on the decomposition of the total variance
of the observations as a sum of the partial variances \( \sigma_m^2 \), \( \sigma_y^2 \), and \( \sigma_r^2 \).

\[
\sigma_m^2 = \frac{N}{11} \sum_j (\bar{X}.j - \bar{X})^2
\]

is the "between months" variance of the transformed
data. It is the column, or treatment, effect that measures the magnitude
of the seasonality, where \( \bar{X}.j = \frac{1}{n} \sum_{i=1}^{N} X_{ij} + I_{1j} \) and \( \bar{X} = \frac{1}{12} \sum_{j} (S + I_{1j}) \).

\[
\sigma_y^2 = \frac{1}{N-1} \sum_i (\bar{X}_i. - \bar{X})^2
\]

is the "between years" variance of the transformed
data. It is the row or block effect that measures the movement of the
seasonality, where \( \bar{X}_i. = \frac{1}{12} \sum_j (S + I_{1j}) \).

\[
\sigma_r^2 = \frac{1}{11(N-1)} \sum_{ij} (|S + I_{1j} - \bar{X}.j - \bar{X}_i. + \bar{X}|)^2
\]

is the "residual" variance.
The null hypothesis $a_1 = a_2 = a_n$, i.e. the one in which the effect of the years in the seasonality does not change can be checked by the ratio $F = \frac{\sigma^2}{\sigma^2_Y}$.

This ratio is related to the Fisher-Snedecor's test with $11$ and $11 (N-1)$ degrees of freedom. The $F$ value is compared with the one in the table for the $F$ test at a 1% level of significance for a series of 10 years of observations (the differences in values for series of different lengths are small).

A table of the analysis of variance is printed at the end of Table D.8 and also a message that indicates whether moving seasonality is present or not.

2. Monthly Moving Seasonality Test for Multiplicative Models

The test is analogous to the previous one except that now the seasonal irregular ratios $SI$ are replaced by $|SI - 100|$.

The moving seasonality tests for quarterly data are analogous to the monthly tests.

The presence of moving seasonality in a series may lead to poor forecasts and large revisions. In some cases, it can be avoided or significantly reduced by switching from the multiplicative to the additive model or vice versa.